## Enhanced TEKS Clarification Document

 Mathematics - Algebraic Reasoning
## ALgebraic Reasoning

§111.48. Implementation of Texas Essential Knowledge and Skills for Mathematics, High School, Adopted 2015.
Source: The provisions of this §111.48 adopted to be effective May 31, 2015, 40 TexReg 3146.
§111.48. Algebraic Reasoning, Adopted 2015 (One Credit).
(a) General requirements. Students shall be awarded one credit for successful completion of this course. Prerequisite: Algebra I.
(b) Introduction.
(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
(3) In Algebraic Reasoning, students will build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I, continue with the development of mathematical reasoning related to algebraic understandings and processes, and deepen a foundation for studies in subsequent mathematics courses. Students will broaden their knowledge of functions and relationships, including linear, quadratic, square root, rational, cubic, cube root, exponential, absolute value, and logarithmic functions. Students will study these functions through analysis and application that includes explorations of patterns and structure, number and algebraic methods, and modeling from data using tools that build to workforce and college readiness such as probes, measurement tools, and software tools, including spreadsheets.
(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

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| AR. 1 | Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to: |
| :---: | :---: |
| AR.1A | Apply mathematics to problems arising in everyday life, society, and the workplace. <br> Apply <br> MATHEMATICS TO PROBLEMS ARISING IN EVERYDAY LIFE, SOCIETY, AND THE WORKPLACE <br> Including, but not limited to: <br> - Mathematical problem situations within and between disciplines <br> - Everyday life <br> - Society <br> - Workplace <br> Note(s): <br> - The mathematical process standards may be applied to all content standards as appropriate. <br> - TxCCRS: <br> - VII.D. Problem Solving and Reasoning - Real-world problem solving <br> - VII.D.1. Interpret results of the mathematical problem in terms of the original real-world situation. <br> - IX.A. Connections - Connections among the strands of mathematics <br> - IX.A.1. Connect and use multiple key concepts of mathematics in situations and problems. <br> - IX.A.2. Connect mathematics to the study of other disciplines. <br> - IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life <br> - IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations. <br> - IX.B.2. Understand and use appropriate mathematical models in the natural, physical, and social sciences. <br> - IX.B.3. Know and understand the use of mathematics in a variety of careers and professions. |
| AR.1B | Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. <br> Use <br> A PROBLEM-SOLVING MODEL THAT INCORPORATES ANALYZING GIVEN INFORMATION, FORMULATING A PLAN OR STRATEGY, DETERMINING A SOLUTION, JUSTIFYING THE SOLUTION, AND EVALUATING THE PROBLEM-SOLVING PROCESS AND THE REASONABLENESS OF THE SOLUTION <br> Including, but not limited to: <br> - Problem-solving model |

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Algebraic Reasoning

- Analyze given information
- Formulate a plan or strategy
- Determine a solution
- Justify the solution
- Evaluate the problem-solving process and the reasonableness of the solution

Note(s):

- The mathematical process standards may be applied to all content standards as appropriate.

TxCCRS

- I.B. Numeric Reasoning - Number sense and number concepts
- I.B.1. Use estimation to check for errors and reasonableness of solutions
- V.A. Statistical Reasoning - Design a study
- V.A.1. Formulate a statistical question, plan an investigation, and collect data.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VII.A.2. Formulate a plan or strategy.
- VII.A.3. Determine a solution.
- VII.A.4. Justify the solution.
- VII.A.5. Evaluate the problem-solving process
- VII.D. Problem Solving and Reasoning - Real-world problem solving
- VII.D.2. Evaluate the problem-solving process.

Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

Select
TOOLS, INCLUDING REAL OBJECTS, MANIPULATIVES, PAPER AND PENCIL, AND TECHNOLOGY AS APPROPRIATE, AND TECHNIQUES, INCLUDING MENTAL MATH, ESTIMATION, AND NUMBER SENSE AS APPROPRIATE, TO SOLVE PROBLEMS

Including, but not limited to:

- Appropriate selection of tool(s) and techniques to apply in order to solve problems
- Tools
- Real objects
- Manipulatives
- Paper and pencil
- Technology
- Techniques

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Algebraic Reasoning

- Mental math
- Estimation
- Number sense

Note(s):

- The mathematical process standards may be applied to all content standards as appropriate
- TxCCRS:
- I.B. Numeric Reasoning - Number sense and number concepts
- I.B.1. Use estimation to check for errors and reasonableness of solutions.
- V.C. Statistical Reasoning - Analyze, interpret, and draw conclusions from data
- V.C.2. Analyze relationships between paired data using spreadsheets, graphing calculators, or statistical software.

Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

Communicate
MATHEMATICAL IDEAS, REASONING, AND THEIR IMPLICATIONS USING MULTIPLE REPRESENTATIONS, INCLUDING SYMBOLS, DIAGRAMS, GRAPHS, AND LANGUAGE AS APPROPRIATE

Including, but not limited to:

- Mathematical ideas, reasoning, and their implications
- Multiple representations, as appropriate
- Symbols
- Diagrams
- Graphs
- Language

Note(s):

- The mathematical process standards may be applied to all content standards as appropriate.
- TxCCRS:
- II.D. Algebraic Reasoning - Representing relationships
- II.D.1. Interpret multiple representations of equations, inequalities, and relationships.
- II.D.2. Convert among multiple representations of equations, inequalities, and relationships.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
- VIII.A.2. Use mathematical language to represent and communicate the mathematical concepts in a problem.


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## Algebraic REASONING

- VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
- VIII.B.2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.
- VIII.C.2. Create and use representations to organize, record, and communicate mathematical ideas.
- VIII.C.3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.

Create and use representations to organize, record, and communicate mathematical ideas.
Create, Use

## REPRESENTATIONS TO ORGANIZE, RECORD, AND COMMUNICATE MATHEMATICAL IDEAS

Including, but not limited to:

- Representations of mathematical ideas
- Organize
- Record
- Communicate
- Evaluation of the effectiveness of representations to ensure clarity of mathematical ideas being communicated
- Appropriate mathematical vocabulary and phrasing when communicating mathematical ideas

Note(s):

- The mathematical process standards may be applied to all content standards as appropriate.
- TxCCRS:
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
- VIII.B.2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.
- VIII.C.2. Create and use representations to organize, record, and communicate mathematical ideas.


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Analyze
MATHEMATICAL RELATIONSHIPS TO CONNECT AND COMMUNICATE MATHEMATICAL IDEAS
Including, but not limited to:

- Mathematical relationships
- Connect and communicate mathematical ideas
- Conjectures and generalizations from sets of examples and non-examples, patterns, etc.
- Current knowledge to new learning

Note(s):

- The mathematical process standards may be applied to all content standards as appropriate.
- TxCCRS:
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
- VIII.A.2. Use mathematical language to represent and communicate the mathematical concepts in a problem.
- VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.
- VIII.C.2. Create and use representations to organize, record, and communicate mathematical ideas.
- VIII.C.3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.
- IX.A. Connections - Connections among the strands of mathematics
- IX.A.1. Connect and use multiple key concepts of mathematics in situations and problems.
- IX.A.2. Connect mathematics to the study of other disciplines.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
Display, Explain, Justify
MATHEMATICAL IDEAS AND ARGUMENTS USING PRECISE MATHEMATICAL LANGUAGE IN WRITTEN OR ORAL COMMUNICATION
Including, but not limited to:
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## Algebraic REASONING

- Finite differences - a list of differences between the $n^{\text {th }}$ successive dependent values, $\Delta y$, when differences between first successive independent values, $\Delta x$, are constant
- $\Delta x$ - change in successive independent values ( $x$-values)
- $\Delta y$ - change in successive dependent values ( $y$-values)
- Differences in values for successive table rows
- Ex:

| $\Delta x$ <br> Independent Finite Difference |  |  | $\Delta y$ |
| :---: | :---: | :---: | :---: |
|  | $x$ | $y$ | Dependent |
|  | -11 | -1337 | Finite Difference |
| $-3-(-7)=4$ | -7 | -349 | -1337) = 988 |
|  | -3 | -33 | $-33-(-349)=316$ |
| $1-(-3)=4$ | 1 | -5 | $-5-(-33)=28$ |
| $5-1=4$ | 5 | 119 | $119-(-5)=124$ |

- Common difference - a common constant finite difference
- Ex:

- Linear functions
- Patterns in $y$-values have a common first difference
- First differences - a list of common differences between the first successive dependent values, $\Delta y$, when first differences between successive independent values, $\Delta x$, are also a common difference
- Linear functions have a dependent first common difference when there is an independent first common difference.
- Ex:


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## Algebraic Reasoning

Sample response:


In both tables, a constant finite difference or common difference results from the second differences between successive $y$-values, $\Delta y$, when the first differences between the successive $x$-values, $\Delta x$, are also constant. Quadratic functions have a dependent second common difference when there is an independent first common difference.

- Cubic functions
- Patterns in $y$-values have a non-zero common third difference
- Third differences - a list of common differences between the third successive $y$-values, $\Delta y$, when first differences between successive $x$ values, $\Delta x$, are also a common difference
- Cubic functions have a dependent third common difference when there is an independent first common difference.
- Ex:


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## ALgebraic Reasoning

The tables show two cubic relationships between $x$ and $y$ in which $y$ is a function of $x$.

Table 1

Table 2


Determine the pattern in finite differences that shows they are cubic functions.

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## Algebraic Reasoning

Sample response:

Table 1

$\Delta y$ $\begin{array}{cc}\text { Dependent } & \text { Dependent } \\ \text { First Difference } & \text { Second Difference }\end{array}$

$$
9-8=1
$$

$25-14=11$
$44-25=19$
$73-44=29$

## 



Dependent Third Common Difference

$$
\begin{gathered}
6-4=2 \\
8-6=2 \\
10-8=2
\end{gathered}
$$

Table $2 \quad \Delta x$


Dependent First Difference

Sample response:
In both tables, a constant finite difference or common difference results from the third differences between successive $y$-values, $\Delta y$, when the first differences between the successive $x$-values, $\Delta x$, are also constant. Cubic functions have a dependent third common difference when there is an independent first common difference.

- Exponential functions
- Patterns in $y$-values differences have a common absolute value resulting in a common ratio
- Common ratio - the constant ratio, $\frac{y_{n}}{y_{n-1}}, y_{n-1} \neq 0$, of successive $y$-values for successive $x$-values, when $\Delta x=1$


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## Algebraic Reasoning

- Common ratio is the same for dependent values and $n$th level differences in the dependent values.
- Exponential functions have successive $y$-value differences that result in a common ratio when there is an independent first common difference.
- Ex:


Determine the pattern in finite differences that shows they are exponential functions.
Sample response:

Table 1


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## Algebraic REASONING

Table $2 \Delta x$

| Independent | $x$ | $y$ |
| :---: | :---: | :---: |
| First Difference | 1 | 500 |
| , | 2 | 250 |
| - | 3 | 125 |
|  | 4 | 62.5 |
|  | 5 | 31.25 |
| $-5=1$ | 6 | 15.625 |


Sample response:
In both tables, the finite differences demonstrate a repeating difference, a common ratio exists between successive $y$-values, $\Delta y$, when the first differences between the successive $x$-values, $\Delta x$, are constant. Exponential functions have successive $y$-values differences that result in a common ratio when there is an independent first common difference.

## Note(s):

- Grade Level(s):
- Grade 8 introduced the formal definition of a linear function.
- Algebra I extended linear functions and introduces quadratic and exponential functions as well as arithmetic and geometric sequences.
- Algebraic Reasoning introduces cubic functions, finite differences, and common ratios.
- Algebra ll will expand on transformations and applications of exponential functions.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- II.D. Algebraic Reasoning - Representing relationships
- II.D.1. Interpret multiple representations of equations, inequalities, and relationships.
- VI.A. Functions - Recognition and representation of functions
- VI.A.2. Recognize and distinguish between different types of functions.
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions.
- VI.C. Functions - Model real-world situations with functions


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## Algebraic Reasoning

- VI.C.1. Apply known functions to model real-world situations.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VII.B. Problem Solving and Reasoning - Proportional reasoning
- VII.B.1. Use proportional reasoning to solve problems that require fractions, ratios, percentages, decimals, and proportions in a variety of contexts using multiple representations.
- VII.C. Problem Solving and Reasoning - Logical reasoning
- VII.C.2. Understand attributes and relationships with inductive and deductive reasoning.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
- VIII.B.2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.


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## Algebraic Reasoning

- Cubic functions have a dependent third common difference when there is an independent first common difference.
- Exponential functions
- Patterns in $y$-values differences have a common absolute value resulting in a common ratio
- Common ratio - the constant ratio, $\frac{y_{n}}{y_{n-1}}, y_{n-1} \neq 0$, of successive $y$-values for successive $x$-values, when $\Delta x=1$
- Common ratio is the same for dependent values and $n$th level differences in the dependent values.
- Exponential functions have finite dependent differences that result in a common ratio when there is an independent first common difference.
- Functions can be classified using finite differences or common ratios.
- Ex:

Classify the relationship as a linear, quadratic, cubic, or exponential function

Sample response:


The relationship represents a linear function because the dependent first differences have a common difference when the independent first differences have a common difference.

- Ex:

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 difference when the independent first differences have a common difference.

- Ex:


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|  | Algebraic Reasoning |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
|  | Classify the relationship as a linear, quadratic, cubic, or exponential function. <br> Sample response: <br> $\Delta x$ Independent <br> First Common Difference <br> Dependent Second Common Difference $8-5=3$ $13-8=5$ $20-13=7$ $29-20=9$ <br> The relationship represents a quadratic function because the dependent second differences result in a common difference when there is an independent first common difference. |  |  |  |  |  |
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## Algebraic ReAsoning

Note(s):

- Grade Level(s):
- Grade 8 introduced the formal definition of a linear function.
- Algebra I extended linear functions and introduces quadratic and exponential functions as well as arithmetic and geometric sequences.
- Algebraic Reasoning introduces cubic functions, finite differences, and common ratios.
- Algebra Il will expand on transformations and applications of exponential functions.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- II.D. Algebraic Reasoning - Representing relationships
- II.D.1. Interpret multiple representations of equations, inequalities, and relationships.
- VI.A. Functions - Recognition and representation of functions
- VI.A.2. Recognize and distinguish between different types of functions.
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions.
- VI.C. Functions - Model real-world situations with functions
- VI.C.1. Apply known functions to model real-world situations.
- VII.B. Problem Solving and Reasoning - Proportional reasoning
- VII.B.1. Use proportional reasoning to solve problems that require fractions, ratios, percentages, decimals, and proportions in a variety of contexts using multiple representations.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
- VIII.B.2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.


## AR.2C

Determine the function that models a given table of related values using finite differences and its restricted domain and range.
Determine
THE FUNCTION THAT MODELS A GIVEN TABLE OF RELATED VALUES USING FINITE DIFFERENCES AND ITS RESTRICTED DOMAIN AND RANGE

Including, but not limited to:

- Finite differences - a list of differences between the $n^{\text {th }}$ successive dependent values, $\Delta y$, when differences between first successive independent values, $\Delta x$, are constant


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## Algebraic Reasoning

- $\Delta x$ - change in successive independent values ( $x$-values)
- $\Delta y$ - change in successive dependent values ( $y$-values)
- Differences in values for successive table rows
- Ex:

| $\Delta x$ Independent Finite Difference$-7-(-11)=4$ |  |  |  |
| :---: | :---: | :---: | :---: |
|  | $x$ | $y$ |  |
|  | -11 | -1337 |  |
|  | -7 | -349 | $\Delta y$ Dependent Finite Difference$\begin{array}{r} -349-(-1337)=988 \\ -33-(-349)=316 \\ -5-(-33)=28 \\ 119-(-5)=124 \end{array}$ |
| $-3-(-7)=4$ | -3 | -33 |  |
| $1-(-3)=4$ | 1 | -5 |  |
|  | 5 | 119 |  |

- Domain and range
- Domain - a set of input values for the independent variable over which the function is defined
- Restricted domain - a set of limited domain values that allows a non-functional relation to become functional
- Range - a set of output values for the dependent variable over which the function is defined
- Notation for domain and range
- $\in$ represents an element of a set
- $\Re$ represents the set of real numbers
- Z represents the set of integers
- Q represents the set of rational numbers
- W represents the set of whole numbers
- N represent the set of natural numbers
- Representations of domain and range
- Verbal description
- Interval notation - notation in which the solution is represented by a continuous interval
- Parentheses indicate that the endpoints are open, meaning the endpoints are excluded from the interval.
- Negative infinity,,$^{\infty}$, and positive infinity, ${ }^{\infty}$, are always associated with a parenthesis.
- Brackets indicate that the endpoints are closed, meaning the endpoints are included in the interval.
- Inequality notation - notation in which the solution is represented by an inequality statement
- Set (builder) notation - notation in which the solution is represented by a set of values
- Braces are used to enclose the set.
- $\{x \mid x \in$ is read as "The set of $x$ such that $x$ is an element of ..."
- A set could be a list of values contained within braces; e.g., $\{1,2,3,4,5\}$.
- Ex:

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|  | Algebraic Reasoning |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Verbal Description | Interval Notation | Inequality Notation | Set (builder) Notation |
|  | $x$ is all real numbers less than five | $(-\infty, 5)$ | $x<5, x \in \Re$ | $\{x \mid x \in \Re, x<5\}$ |
|  | $x$ is all real numbers | $(-\infty, \infty)$ | $x \in \Re$ | $\{x \mid x \in \mathfrak{R}\}$ |
|  | $x$ is all whole numbers greater than -3 and less than or equal to 6 | $(-3,6], x \in W$ | $-3<x \leq 6, x \in W$ | $\{y \mid x \in W,-3<x \leq 6\}$ |
|  | $y$ is all real numbers greater than -3 and less than or equal to 6 | $(-3,6]$ | $-3<y \leq 6, y \in \Re$ | $\{y \mid y \in \Re,-3<y \leq 6\}$ |
|  | $y$ is all integers greater than or equal to zero | $[0, \infty), y \in \mathrm{Z}$ | $y \geq 0, y \in$ | $\{y \mid y \in \mathrm{Z}, \mathrm{y} \geq 0\}$ |

- Restricted domain and range
- Domain and range values for the function model is limited to given data set.
- Ex:



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- Common difference - a common constant finite difference
- Ex:


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- Quadratic functions
- Degree two polynomial function
- Patterns in $y$-values have a non-zero common second difference
- Second differences - a list of common differences between the second successive dependent values, $\Delta y$, when the differences between first successive independent values, $\Delta x$, are also constant.
- Quadratic functions have a dependent second common difference when there is an independent first common difference.
- Quadratic function representation
- Standard form, $f(x)=a x^{2}+b x+c$, where $a, b$, and $c$ are rational numbers
- First difference between the $y$-values when $x=0$ and $x=1$ is $a+b$.
- Second common differences between the $y$-values are related to a by second common difference is $2 a$ when $\Delta x=1$.
- Value of $c$ represents $(0, c)$ or the $y$-intercept


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## Algebraic Reasoning

Quadratic Functions


Since a second common difference results from the second finite differences between successive $y$-values, $\Delta y$, when the differences between successive $x$-values, $\Delta x$, are also constant, the pattern demonstrates a quadratic function relationship.

Quadratic functions exhibit a pattern of a second common difference for $\Delta y$. Since $\Delta x=1$, there exists a relationship where the second common difference equals 2 a.

- Ex:


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## Algebraic Reasoning

Determine the function that models the given table of related values using finite differences. Identify the domain and range values for the function.

| $x$ | $f(x)$ |
| :---: | :---: |
| -1 | 5 |
| 0 | 4 |
| 1 | 5 |
| 2 | 8 |
| 3 | 13 |

Sample response:


Since a second common difference results from the second finite differences between successive $y$-values, $\Delta y$, when the differences between successive $x$-values, $\Delta x$, are also constant, the pattern demonstrates a quadratic function relationship.

Quadratic functions exhibit a pattern of a second common difference for $\Delta y$. Since $\Delta x=1$, there exists a relationship where the second common difference equals $2 a$.

| Standard form |  |  |
| :--- | :--- | :---: |
| The second common differences | $y=a x^{2}+b x+c$ |  |
| between the $y$-values are related to $a$ | $y=(1) x^{2}+(0) x+4$ |  |

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## Algebraic Reasoning

- Cubic functions
- Degree three polynomial function
- Patterns in $y$-values have a non-zero common third difference
- Third differences - a list of common differences between the third successive $y$-values, $\Delta y$, when first differences between successive $x$ values, $\Delta x$, are also a common difference
- Cubic functions have a dependent third common difference when there is an independent first common difference.
- Third differences that result in a constant difference or a common difference represent a cubic function.
- Cubic function representation
- Standard form, $f(x)=a x^{3}+b x^{2}+c x+d$, where $a, b, c$, and $d$ are rational numbers
- First difference between the $y$-values when $x=0$ and $x=1$ is $a+b+c$.
- Second difference between the first two differences of $y$-values between $x=0$ and $x=1$, and $x=1$ and $x=2$, is $6 a+2 b$
- Third common differences between the $y$-values are related to a by third common difference $=6 a$, when $\Delta x=1$.
- Value of $d$ represents $(0, d)$ or the $y$-intercept


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- Ex:


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## Algebraic ReAsoning

Determine the function that models the given table of related values using finite differences. Identify the domain and range values for the function.

| $x$ | $y$ |
| :---: | :---: |
| -1 | 6 |
| 0 | 5 |
| 1 | 4 |
| 2 | -3 |
| 3 | -22 |

Sample response
$\Delta x$


Since a third common difference results from the third finite differences between successive $y$-values, $\Delta y$, when the differences between successive $x$-values, $\Delta x$, are also constant, the pattern demonstrates a cubic function relationship.

Cubic functions exhibit a pattern of a third common difference for $\Delta y$. Since $\Delta x=1$, there exists a relationship where the third common difference equals $6 a$


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- Exponential functions
- Patterns in $y$-values differences have a common absolute value resulting in a common ratio
- Exponential functions have successive $y$-value differences that result in a common ratio when there is an independent first common difference.
- Standard form, $f(x)=a b^{x}$, where a represents the initial value, or the $y$-intercept $(0, a)$, and $b$ represents the base of the exponential function, or the common ratio
- Common ratio - the constant ratio, $\frac{y_{n}}{y_{n-1}}, y_{n-1} \neq 0$, of successive $y$-values for successive $x$-values, when $\Delta x=1$
- Common ratio is the same for dependent values and $n$th level differences in the dependent values.


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## Algebraic Reasoning

## Exponential Functions


$\Delta y$ Dependent First Difference

$$
a b-a=a(b-1)
$$

$a b(b)-a b=a b(b-1)$
$a b(b)(b)-a b(b)=a b^{2}(b-1)$

$$
a b(b)(b)(b)-a b(b)(b)=a b^{3}(b-1)
$$

$a b(b)(b)(b)(b)-a b(b)(b)(b)=a b^{4}(b-1)$
Repeated Finite Differences

Sample response:
Since the finite differences demonstrate a repeating difference, a common ratio exists between successive $y$-values for successive $x$-values. This pattern demonstrates an exponential function relationship.
Exponential functions exhibit a pattern of constant ratios, $\frac{y_{n}}{y_{n-1}}=\frac{b}{1}=b$.

- Ex:

Determine the function that models the given table of related values using finite differences. Identify the domain and range values for the function.

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| ALGEBRAIC REASONING |  |  |
| :---: | :---: | :---: | :---: |
|  |  | $x$ $y$ <br> 0 1000 <br> 1 500 <br> 2 250 <br> 3 125 <br> 4 62.5 <br> 5 31.25 <br> 6 15.625 |



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## ALgebraic REASONING

- Algebra II will expand on transformations and applications of exponential functions.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS
- II.D. Algebraic Reasoning - Representing relationships
- II.D.1. Interpret multiple representations of equations, inequalities, and relationships.
- II.D.2. Convert among multiple representations of equations, inequalities, and relationships.
- V.C. Statistical Reasoning - Analyze, interpret, and draw conclusions from data
- V.C.2. Analyze relationships between paired data using spreadsheets, graphing calculators, or statistical software
- VI.A. Functions - Recognition and representation of functions
- VI.A.2. Recognize and distinguish between different types of functions.
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions
- VI.C. Functions - Model real-world situations with functions
- VI.C.1. Apply known functions to model real-world situations.
- VI.C.2. Develop a function to model a situation.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
- VIII.B.2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context.
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- VIII.C.1. Communicate mathematicalideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.
- VIII.C.2. Create and use representations to organize, record, and communicate mathematical ideas.
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
- IX.B.2. Understand and use appropriate mathematical models in the natural, physical, and social sciences.

Determine a function that models real-world data and mathematical contexts using finite differences such as the age of a tree and its circumference, figurative numbers, average velocity, and average acceleration.

Determine

A RESULTING FUNCTION THAT MODELS REAL-WORLD DATA AND MATHEMATICAL CONTEXTS USING FINITE DIFFERENCES SUCH AS THE AGE OF A TREE AND ITS CIRCUMFERENCE, FIGURATIVE NUMBERS, AVERAGE VELOCITY, AND AVERAGE ACCELERATION.

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## Algebraic REASONING

## Including, but not limited to:

- Finite differences - a list of differences between the $n^{\text {th }}$ successive dependent values, $\Delta y$, when differences between first successive independent values, $\Delta x$, are constant
- $\Delta x$ - change in successive independent values ( $x$-values)
- $\Delta y$ - change in successive dependent values ( $y$-values)
- Differences in values for successive table rows
- Ex:

| $\Delta x$ Independent Finite Difference $-7-(-11)=4$ | $x$ | $y$ | $\Delta y$ Dependent Finite Difference |
| :---: | :---: | :---: | :---: |
|  | -11 | -1337 |  |
|  |  |  | $-349-(-1337)=988$ |
|  | -7 | -349 |  |
|  | -3 | -33 | $-33-(-349)=316$ |
|  | 1 | -5 | $-5-(-33)=28$ |
|  | 5 | 119 | $119-(-5)=124$ |

- Common difference - a common constant finite difference
- Ex:

- Linear functions
- Degree one polynomial function
- Patterns in $y$-values have a common first difference
- First differences - a list of common differences between the first successive dependent values, $\Delta y$, when first differences between successive independent values, $\Delta x$, are also a common difference
- Linear functions have a dependent first common difference when there is an independent first common difference.


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- Linear function representations
- Standard form, $a x+b y=c$
- Slope-intercept form, $y=m x+b$, where $m$ represents the slope and $b$ represents the $y$-intercept
- Slope of a linear function is represented by $m=\frac{\text { change in } y \text {-values }}{\text { change in } x \text {-values }}=\frac{\Delta y}{\Delta x}=\frac{\text { first common difference }}{\Delta x}$.
- $(x, y)$ represents a point on the line
- brepresents the $y$-intercept, $(0, b)$
- Point-slope form, $y-y_{1}=m\left(x-x_{1}\right)$, where $m$ is the slope and ( $x_{1}, y_{1}$ ) represents a point of the line
- Slope of a linear function is represented by $m=\frac{\text { change in } y \text {-values }}{\text { change in } x \text {-values }}=\frac{\Delta y}{\Delta x}=\frac{\text { first common difference }}{\Delta x}$.
- $\left(x_{1}, y_{1}\right)$ represents a point on the line
- Quadratic functions
- Degree two polynomial function
- Patterns in $y$-values have a non-zero common second difference
- Second differences - a list of common differences between the second successive dependent values, $\Delta y$, when the differences between first successive independent values, $\Delta x$, are also constant.
- Quadratic functions have a dependent second common difference when there is an independent first common difference.
- Quadratic function representation
- Standard form, $f(x)=a x^{2}+b x+c$, where $a, b$, and $c$ are rational numbers
- First difference between the $y$-values when $x=0$ and $x=1$ is $a+b$.
- Second common differences between the $y$-values are related to a by second common difference is $2 a$ when $\Delta x=1$.
- Value of $c$ represents $(0, c)$ or the $y$-intercept
- Cubic functions
- Degree three polynomial function
- Patterns in $y$-values have a non-zero common third difference
- Third differences - a list of common differences between the third successive $y$-values, $\Delta y$, when first differences between successive $x$ values, $\Delta x$, are also a common difference
- Cubic functions have a dependent third common difference when there is an independent first common difference.
- Third differences that result in a constant difference or a common difference represent a cubic function.
- Cubic function representation
- Standard form, $f(x)=a x^{3}+b x^{2}+c x+d$, where $a, b, c$, and $d$ are rational numbers
- First difference between the $y$-values when $x=0$ and $x=1$ is $a+b+c$.
- Second difference between the first two differences of $y$-values between $x=0$ and $x=1$, and $x=1$ and $x=2$, is $6 a+2 b$
- Third common differences between the $y$-values are related to a by third common difference $=6 a$, when $\Delta x=1$.
- Value of $d$ represents $(0, d)$ or the $y$-intercept
- Exponential functions


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## Algebraic Reasoning

- Patterns in $y$-values differences have a common absolute value resulting in a common ratio
- Exponential functions have successive $y$-value differences that result in a common ratio when there is an independent first common difference.
- Standard form, $f(x)=a b^{x}$, where a represents the initial value, or the $y$-intercept ( $0, a$ ), and $b$ represents the base of the exponential function, or the common ratio
- Common ratio - the constant ratio, $\frac{y_{n}}{y_{n-1}}, y_{n-1} \neq 0$, of successive $y$-values for successive $x$-values, when $\Delta x=1$
- Common ratio is the same for dependent values and $n$th level differences in the dependent values.
- Real-world data and mathematical contexts
- Age of a tree and its circumference
- Ex:

A scientist was studying tree growth in a national forest. She was counting tree rings to determine the age of the tree and measuring the circumference of the rings to predict future growth. The table lists the age of the tree and the circumference of each ring.

| Age (years) <br> $x$ | 10 | 11 | 12 | 13 | 14 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Circumference (cm) <br> $c(x)$ | 78.5 | 86.35 | 94.2 | 102.05 | 109.9 |

Using finite differences, determine a function to predict tree growth.
Sample response:


## $\Delta y$ <br> Dependent

 First Common Difference| $86.35-78.5$ | $=7.85$ |
| ---: | :--- |
| $94.2-86.35$ | $=7.85$ |
| $102.05-94.2$ | $=7.85$ |
| $109.9-102.05$ | $=7.85$ |

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## Algebraic Reasoning

Since a first common difference results from the first finite differences between successive $y$-values, $\Delta y$, when the differences between successive $x$-values, $\Delta x$, are also constant, the pattern demonstrates a linear function relationship.

Since the relationship represents a linear function, the slope of the linear function is
$\frac{\Delta y}{\Delta x}=\frac{\text { first common difference }}{\Delta x}=\frac{7.85}{1}=7.85$.


The function $c(x)=7.85 x$ can be used to predict tree growth, where $x$ represents the age of the tree in years and $c(x)$ represents the circumference in centimeters of the tree.

- Figurative numbers
- Ex:

The sequence of shapes is a representative of figurative numbers.


Using finite differences, determine a function to model the number of dots for any position of the sequence.

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## Algebraic Reasoning

Sample response:
It is reasonable to include a data point representing that at position 0 there would be 0 dots.


Since a second common difference results from the second finite differences between successive $y$-values, $\Delta y$, when the differences between successive $x$-values, $\Delta x$, are also constant, the pattern demonstrates a quadratic function relationship.

Quadratic functions exhibit a pattern of a second common difference for $\Delta y$. Since $\Delta x=1$, there exists a relationship where the second common difference equals $2 a$.


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## Algebraic Reasoning

Eric drove cross country to his family's home for the holidays. The table represents the distance in miles he traveled with respect to the time in hours that he drove.

| $\begin{gathered} \text { Time (hours) } \\ t \end{gathered}$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \hline \text { Distance (miles) } \\ d(t) \end{gathered}$ | 64.7 | 129.4 | 194.1 | 258.8 | 323.5 |

Using finite differences, determine a function that models the distance in miles he traveled with respect to the time in hours that he drove.

## Sample response:

It is reasonable to include a data point representing that at 0 hours Eric had driven 0 miles


Since a first common difference results from the first finite differences between successive $y$-values, $\Delta y$, when the differences between successive $x$-values, $\Delta x$, are also constant, the pattern demonstrates a linear function relationship.

Since the relationship represents a linear function, the slope of the linear function is
$\frac{\Delta y}{\Delta x}=\frac{\text { first common difference }}{\Delta x}=\frac{64.7}{1}=64.7$.


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## Algebraic Reasoning

Sample response:
It is reasonable to include a data point representing that at 0 seconds there was 0 acceleration.


Since a second common difference results from the second finite differences between successive $y$-values, $\Delta y$, when the differences between successive $x$-values, $\Delta x$, are also constant, the pattern demonstrates a quadratic function relationship.

Quadratic functions exhibit a pattern of a second common difference for $\Delta y$. Since $\Delta x=1$, there exists a relationship where the second common difference equals $2 a$.

| Standard Form |  |
| :--- | :--- |
| The second common differences <br> between the $y$-values are related to $a$ <br> by second common difference <br> se | $y=a x^{2}+b x+c$ |
| $\frac{20}{1}=2 a$ | $y=(10) x^{2}+(8) x+0$ |
| $2 a=20$ |  |
| $a=10$ | $y=10 x^{2}+8 x$ |
| The first difference between the |  |
| $y$ values when $x=0$ and $x=1$ is $a+b$. |  |

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## ALgebraic ReAsoning

- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VII.D. Problem Solving and Reasoning - Real-world problem solving
- VII.D.1. Interpret results of the mathematical problem in terms of the original real-world situation.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
- VIII.B.2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.
- VIII.C.2. Create and use representations to organize, record, and communicate mathematical ideas.
- IX.A. Connections - Connections among the strands of mathematics
- IX.A.2. Connect mathematics to the study of other disciplines.
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
- IX.B.2. Understand and use appropriate mathematical models in the natural, physical, and social sciences.

Compare and contrast the key attributes, including domain, range, maxima, minima, and intercepts, of a set of functions such as a se comprised of a linear, a quadratic, and an exponential function or a set comprised of an absolute value, a quadratic, and a square root function tabularly, graphically, and symbolically.

Compare and Contrast
THE KEY ATTRIBUTES, INCLUDING DOMAIN, RANGE, MAXIMA, MINIMA, AND INTERCEPTS, OF A SET OF FUNCTIONS SUCH AS A SET COMPRISED OF A LINEAR, A QUADRATIC, AND AN EXPONENTIAL FUNCTION OR A SET COMPRISED OF AN ABSOLUTE VALUE, A QUADRATIC, AND A SQUARE ROOT FUNCTION TABULARLY, GRAPHICALLY, AND SYMBOLICALLY

Including, but not limited to:

- Function - a relation in which each element of the domain $(x)$ is paired with exactly one element of the range ( $y$ )
- Key attributes of functions
- Domain and range
- Domain - a set of input values for the independent variable over which the function is defined
- Restricted domain - a set of limited domain values that allows a non-functional relation to become functional

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## Algebraic Reasoning

- Range - a set of output values for the dependent variable over which the function is defined
- Notation for domain and range
- $\in$ represents an element of a set
- $\Re$ represents the set of real numbers
- Z represents the set of integers
- Q represents the set of rational numbers
- W represents the set of whole numbers
- $N$ represent the set of natural numbers
- Representations of domain and range
- Verbal description
- Interval notation - notation in which the solution is represented by a continuous interval
- Parentheses indicate that the endpoints are open, meaning the endpoints are excluded from the interval.
- Negative infinity, $-\infty$, and positive infinity, $\infty$, are always associated with a parenthesis.
- Brackets indicate that the endpoints are closed, meaning the endpoints are included in the interval.
- Inequality notation - notation in which the solution is represented by an inequality statement
- Set (builder) notation - notation in which the solution is represented by a set of values
- Braces are used to enclose the set.
- $\{x \mid x \in$ is read as "The set of $x$ such that $x$ is an element of ..."
- A set could be a list of values contained within braces; e.g., $\{1,2,3,4,5\}$.
- Ex:

| Verbal <br> Description | Interval <br> Notation | Inequality <br> Notation | Set (builder) <br> Notation |
| :--- | :---: | :---: | :---: |
| $x$ is all real numbers <br> less than five | $(-\infty, 5)$ | $x<5, x \in \Re$ | $\{x \mid x \in \Re, x<5\}$ |
| $x$ is all real numbers | $(-\infty, \infty)$ | $x \in \Re$ | $\{x \mid x \in \Re\}$ |
| $x$ is all whole numbers <br> greater than -3 and <br> less than or equal to 6 | $(-3,6], x \in \mathrm{~W}$ | $-3<x \leq 6, x \in \mathrm{~W}$ | $\{y \mid x \in \mathrm{~W},-3<x \leq 6\}$ |
| $y$ is all real numbers <br> greater than -3 and <br> less than or equal to 6 | $(-3,6]$ | $-3<y \leq 6, y \in \Re$ | $\{y \mid y \in \Re,-3<y \leq 6\}$ |
| $y$ is all integers <br> greater than or equal <br> to zero | $[0, \infty), y \in \mathrm{Z}$ | $y \geq 0, y \in \mathrm{Z}$ | $\{y \mid y \in \mathrm{Z}, y \geq 0\}$ |

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## Mathematics Enhanced TEKS Clarification Document

## Algebraic REASONING

- Asymptote - a line that is approached and may or may not be crossed
- Horizontal asymptote - horizontal line approached by the curve as the function approaches positive or negative infinity. Horizontal asymptotes may be crossed by the curve.
- Exponential and rational functions have horizontal asymptotes
- Vertical asymptote - vertical line approached by the curve as the function approaches positive or negative infinity. Vertical asymptotes are never crossed by the curve.
- Logarithmic and rational functions have vertical asymptotes
- Asymptotic behavior - behavior such that as $x$ approaches infinity, $f(x)$ or $y$ approaches a given value
- Maximum (extremum) - largest $y$-coordinate or $y$-value a function takes over the entire domain of the curve represented as a coordinate set
- Minimum (extremum) - smallest $y$-coordinate or $y$-value a function takes over the entire domain of the curve represented as a coordinate set
- $x$-intercept $-x$-coordinate of a point at which the relation crosses the $x$-axis, meaning the $y$-coordinate equals zero, $(x, 0)$
- Zeros - the value(s) of $x$ such that the $y$-value of the relation equals zero
- Linear functions have at most one $x$-intercept
- Polynomial functions may have up to the degree number of $x$-intercepts
- Exponential, logarithmic, rational, cube root, and square root functions have at most one $x$-intercept
- Absolute value functions involving the absolute value of a linear expression may have zero, one, or two $x$-intercepts
- $y$-intercept $-y$-coordinate of a point at which the relation crosses the $y$-axis, meaning the $x$-coordinate equals zero, ( $0, y$ )
- Functions with a domain of all real numbers have one $y$-intercept.
- Functions where the dependent value is undefined when the independent value is 0 will not have a $y$-intercept.
- Sets of functions
- Set of a linear, quadratic, and exponential function
- Ex:


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## Algebraic Reasoning

## Symbolically:

- $f(x)=2 x+3$, solve for $x$ when $f(x)=$ 0

$$
0=2 x+3
$$

$$
0-3=2 x+3-3
$$

$$
-3=2 x
$$

| $y$-intercept |  |  |
| :---: | :---: | :---: |
| Table and Graph: <br> - $y$-intercept of 3 located at the point $(0,3)$. | Table and Graph: <br> - $y$-intercept of -6 located at the point $(0,-6)$. | Table and Graph: <br> - $y$-intercept of 3 located at the point $(0,3)$. |
| Symbolically: <br> - $f(x)=2 x+3$, evaluate $f(x)$ when $x=$ 0 | Symbolically: $\begin{aligned} & g(x)=x^{2}-x-6 \text {, evaluate } g(x) \text { when } x \\ & =0 \end{aligned}$ | Symbolically: <br> $h(x)=3(0.5)^{x}$, evaluate $h(x)$ when $x=0$ |
| $\begin{aligned} & f(0)=2(0)+3 \\ & f(0)=0+3 \\ & f(0)=3 \end{aligned}$ | $\begin{aligned} & g(0)=(0)^{2}-(0)-6 \\ & g(0)=0-0-6 \\ & g(0)=-6 \end{aligned}$ | $\begin{aligned} & h(0)=3(0.5)^{(0)} \\ & h(0)=3(1) \\ & h(0)=3 \end{aligned}$ |
| The $y$-intercept is 3 . | The $y$-intercept is -6 . | The $y$-intercept is 3 . |
| Maximum and Minimum |  |  |
| Table and Graph: <br> - Patterns in table and graph indicate $f(x)$ has no maximum or minimum | Table and Graph: <br> - Minimum of -6.25 , located at the vertex, (0, -6.25) | Table and Graph: <br> - Patterns in table and graph indicate $h(x)$ has no maximum or minimum |
| Symbolically: <br> - $f(x)=2 x+3$, substituting larger and larger positive $x$-values results in | Symbolically: <br> - $g(x)=x^{2}-x-6$, substituting larger and larger $x$-values or smaller and | Symbolically: <br> - $h(x)=3(0.5)^{x}$, substituting larger and larger positive $x$-values results in |

- $h(x)=3(0.5)^{x}$, evaluate $h(x)$ for negative values, 0 , and positive values of $x$. All values of $x$, as indicated by the table and graph, result in positive
$y$-values; therefore, $h(x)=3(0.5)^{x}$ has no $x$-intercept because $h(x)$ never equals zero.

$$
\frac{-3}{2}=\frac{2 x}{2}
$$

## Symbolically:

- $g(x)=x^{2}-x-6$, solve for $x$ when $g(x)=0$
$0=x^{2}-x-6$
$0=(x-3)(x+2)$
$0=x-3$ and $0=x+2$
$3=x$ and $-2=x$
The $x$-intercepts are 3 and -2 .

$$
-\frac{3}{2}=x
$$

$$
-1.5=x
$$

The $x$-intercept is -1.5 .
$y$-intercept of 3 located at the point

Symbolically:
$h(x)=3(0.5)^{x}$, evaluate $h(x)$ when
$h(0)=3(0.5)^{(0)}$
$h(0)=3(1)$
$h(0)=3$

## Graph:

Patterns in table and graph indicate Symbolically:
$h(x)=3(0.5)^{x}$, subs larger positive $x$-values results in

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## Algebraic Reasoning



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## Algebraic Reasoning

## Table and Graph:

- Patterns in table and graph indicate $f(x)$ has no vertical or horizontal asymptotes

Symbolically:

- $f(x)=2 x+3$ does not approach a particular $y$-value as the $x$-values get larger and larger in the positive direction (or smaller and smaller in the negative direction); therefore $f(x)$ has no horizontal asymptote
- $f(x)=2 x+3$ does not get larger and larger (or smaller and smaller) as the $x$-values approach a particular number; therefore $f(x)$ has no vertical asymptote


## Table and Graph:

- Patterns in table and graph indicate $f(x)$ has no vertical or horizontal asymptotes

Symbolically:

- $g(x)=x^{2}-x-6$ does not approach a particular $y$-value as the $x$-values get larger and larger in the positive direction (or smaller and smaller in the negative direction); therefore $g(x)$ has no horizontal asymptote
- $g(x)=x^{2}-x-6$ does not get larger and larger (or smaller and smaller) as the $x$-values approach a particular number; therefore $g(x)$ has no vertical asymptote


## Table and Graph:

- Patterns in table and graph indicate $h(x)$ has no vertical asymptote
- $h(x)$ has a horizontal asymptote at the line $y=0$

Symbolically:

- $h(x)=3(0.5)^{x}$ approaches the $y$-value of 0 as the $x$-values get larger and larger in the positive direction; therefore $h(x)$ has a horizontal asymptote at the line $y=0$
- $h(x)=3(0.5)^{x}$ does not get larger and larger (or smaller and smaller) as the $x$-values approach a particular number; therefore $h(x)$ has no vertical asymptote

| Table and Graph: <br> - $\{-3,-2,-1.5,-1,0,1,2,3,4\}$ <br> Graphically and Symbolically: <br> - All real numbers $\begin{aligned} & x \in \mathfrak{R} \\ & \{x \mid x \in \mathfrak{R}\} \\ & (-\infty, \infty) \end{aligned}$ | Table and Graph: <br> - $\{-3,-2,-1,0,0.5,1,2,3,4\}$ <br> Graphically and Symbolically: <br> - All real numbers <br> $x \in \Re$ <br> $\{x \mid x \in \mathfrak{R}\}$ | Table and Graph: <br> - $\{-3,-2,-1,0,1,2,3,4,5\}$ <br> Graphically and Symbolically: <br> - All real numbers $\begin{aligned} & x \in \mathfrak{R} \\ & \{x \mid x \in \mathfrak{R}\} \\ & (-\infty, \infty) \end{aligned}$ |
| :---: | :---: | :---: |
| Table and Graph: <br> - $\{-3,-1,0,1,3,5,7,9,11\}$ <br> Graphically and Symbolically: <br> - All real numbers $\begin{aligned} & y \in \mathfrak{R} \\ & \{y \mid y \in \mathfrak{R}\} \\ & (-\infty, \infty) \end{aligned}$ <br> Table and Graph: <br> $\{6,0,-4,-6,-6.25,-6,-4,0,6\}$ <br> Graphically and Symbolically: <br> - All real numbers greater than or equal to -6.25 <br> $y \geq-6.25$ <br> $\{y \mid y \in \mathfrak{R}, y \geq-6.25\}$ <br> $[-6.25, \infty)$ |  |  |
|  |  | Table and Graph: <br> - $\{24,12,6,3,1.5,0.75,0.375,0.1825$, $0.09375\}$ <br> Graphically and Symbolically: <br> - All real numbers greater than 0 $\begin{aligned} & y>0 \\ & \{y \mid y \in \mathfrak{R}, y>0\} \\ & (0, \infty) \end{aligned}$ |
| Summary of key atributes of $f(x)=2 x+3, g(x)=x^{2}-x-6$, and $h(x)=3(0.5)^{x}$ |  |  |

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## Algebraic REASONING

| Attribute | $f(x)=2 x+3$ | $g(x)=x^{2}-x-6$ | $h(x)=3(0.5)^{x}$ |
| :---: | :---: | :---: | :---: |
| $x$-intercept(s) | $(-1.5,0)$ | $(-2,0)$ and $(3,0)$ | none |
| $y$-intercept | $(0,3)$ | $(0,-6)$ | $(0,3)$ |
| Maximum | none | none | none |
| Minimum | none | (0, -6.25) | none |
| Horizontal asymptote | none | none | $y=0$ |
| Vertical asymptote | none | none | none |
| Domain | All real numbers $\begin{gathered} x \in \mathfrak{R} \\ \{x \mid x \in \mathfrak{R}\} \\ (-\infty, \infty) \end{gathered}$ | All real numbers $\begin{gathered} x \in \Re \\ \{x \mid x \in \mathfrak{R}\} \\ (-\infty, \infty) \end{gathered}$ | All real numbers $\begin{gathered} x \in \mathfrak{R} \\ \{x \mid x \in \mathfrak{R}\} \\ (-\infty, \infty) \end{gathered}$ |
| Range | All real numbers $\begin{gathered} y \in \mathfrak{R} \\ \{y \mid y \in \mathfrak{R}\} \\ (-\infty, \infty) \end{gathered}$ | All real numbers greater than or equal to -6.25 $\begin{gathered} y \geq-6.25 \\ \{y \mid y \in \mathfrak{R}, y \geq-6.25\} \\ {[-6.25, \infty)} \end{gathered}$ | All real numbers greater than zero $\begin{gathered} y>0 \\ \{y \mid y \in \mathfrak{R}, y>0\} \\ (0, \infty) \\ \hline \end{gathered}$ |

- Set of an absolute value, quadratic, and square root function
- Ex:

Given $f(x)=2|x-3|-4, g(x)=-2 x^{2}+8 x-8$, and $h(x)=\sqrt{x-2}+3$, compare and contrast the key attributes of $f(x)$, $g(x)$, and $h(x)$.

$$
f(x)=2|x-3|-4
$$

$$
h(x)=\sqrt{x-2}+3
$$

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## Algebraic Reasoning

> - $f(x)=2|x-3|-4$, solve for $x$ when $f(x)=0$
> Since $0=2|x-3|-4$, then
> $|x-3|$ would equal 2 because $0=2(2)-4$. Since $|1-3|$ and $|5-3|$ both equal 2 , then $x=1$ and $x=5$.

The $x$-intercepts are 1 and 5 , located at points $(1,0)$ and $(5,0)$.

$$
\begin{aligned}
& g(x)=-2 x^{2}+8 x-8, \text { solve for } x \text { when } \\
& g(x)=0 \\
& g(x)=-2 x^{2}+8 x-8 \\
& 0=-2 x^{2}+8 x-8 \\
& 0=-2\left(x^{2}-4 x+2\right) \\
& 0=-2(x-2)(x-2) \\
& 0 \neq-2 \text { and } 0=x-2 \text { and } 0=x-2 \\
& 2=x \text { and } 2=x
\end{aligned}
$$

The $x$-intercept is 2 , located at the point (2, 0).

- $h(x)=\sqrt{x-2}+3$, substitute positive values larger than or equal to 2 for $x$, and solve for $y$.
- All values, as indicated by the table and graph, result in positive $y$-values; therefore, $h(x)$ has no $x$-intercept.

Table and Graph:

- $y$-intercept of 2 located at the point $(0,2)$

Symbolically:

- $f(x)=2|x-3|-4$, evaluate $f(x)$ when $x=0$
$f(x)=2|x-3|-4$
$f(0)=2|(0)-3|-4$
$f(0)=2|-3|-4$
$f(0)=2(3)-4$
$f(0)=6-4$
$f(0)=2$
The $y$-intercept is 2 , located at the point ( 0,2 ).

Table and Graph:

- $y$-intercept of -8 , located at the point $(0,-8)$


## Symbolically:

- $g(x)=-2 x^{2}+8 x-8$, evaluate $g(x)$ when $x=0$
$g(x)=2 x^{2}+8 x-8$
$g(0)=2(0)^{2}+8(0)-8$
$g(0)=2(0)+0-8$
$g(0)=0+0-8$
$g(0)=-8$

Table and Graph:

- no $y$-intercept since the $x$-values never become 0 or negative

Symbolically:

- $h(x)=\sqrt{x-2}+3$, evaluate $h(x)$ when $x=0$

$$
\begin{aligned}
& h(x)=\sqrt{x-2}+3 \\
& h(0)=\sqrt{(0)-2}+3 \\
& h(0)=\sqrt{-2}+3 \\
& h(0)=\text { undefined }+3 \\
& h(0)=\text { undefined }
\end{aligned}
$$

The $y$-intercept is -8 , located at the point (0, -8).
$h(x)$ is undefined at $x=0$; therefore $h(x)$ has no $y$-intercept.

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## Algebraic Reasoning

Table and Graph:

- Minimum of -4 , located at the point $(3,-4)$

Symbolically:

- $f(x)=2|x-3|-4$, substituting larger and larger positive $x$-values results in larger and larger $y$-values
- $f(x)=2|x-3|-4$, substituting smaller and smaller negative $x$-values results in larger and larger $y$-values.

Therefore, $f(x)$ has no maximum.

- Since a minimum value would occur when $|x-3|=0$, then $x=3$.

$$
\begin{aligned}
& f(x)=2|3-3|-4 \\
& f(x)=2(0)-4 \\
& f(x)=-4
\end{aligned}
$$

$f(x)$ has a minimum of -4 , located at the point $(3,-4)$.

Table and Graph:

- Maximum of 0 , located at the vertex, $(2,0)$


## Symbolically:

- $g(x)=-2 x^{2}+8 x-8$, substituting larger and larger positive $x$-values or smaller and smaller negative $x$-values results in smaller and smaller negative $y$-values; therefore $g(x)$ has no minimum
- Use $x=-\frac{b}{2 a}$ when $g(x)=a x^{2}+b x+c$ to determine the $x$-coordinate of the vertex.
$g(x)=-2 x^{2}+8 x-8$
$a=-2, b=8$
$x=-\frac{b}{2 a}=-\frac{(8)}{2(-2)}=-\frac{8}{-4}=2$
Use the $x$-value to determine the $y$-coordinate of the vertex.
$g(2)=-2(2)^{2}+8(2)-8$
$g(2)=-2(4)+16-8$
$g(2)=-8+16-8$
$g(2)=0$
Vertex: $(h, k)=(2,0)$
$g(x)$ has a maximum of 0 , located at the point (2, 0).

Table and Graph

- Minimum of 3 , located at the point $(2,3)$


## Symbolically:

- $h(x)=\sqrt{x-2}+3$, substituting larger and larger positive $x$-values results in larger and larger $y$-values; therefore $h(x)$ has no maximum
- $f(x)=\sqrt{x-2}+3$, substituting $x$-values less than 2 makes $h(x)$ undefined and $h(2)=3$; therefore $h(x)$ has a minimum of 3 , located at the point $(2,3)$
$h(x)$ has a minimum of 3 , located at the point (2, 3).


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## Algebraic Reasoning

## Table and Graph:

- Patterns in table and graph indicate $f(x)$ has no vertical or horizontal asymptotes

Symbolically:

- $f(x)=2|x-3|-4$ does not approach a particular $y$-value as the $x$-values get larger and larger in the positive direction (or smaller and smaller in the negative direction); therefore $f(x)$ has no horizontal asymptote
- $f(x)=2|x-3|-4$ does not get larger and larger (or smaller and smaller) as the $x$-values approach a particular number; therefore $f(x)$ has no vertical asymptote


## Table and Graph:

- Patterns in table and graph indicate $g(x)$ has no vertical or horizontal asymptotes


## Symbolically:

- $g(x)=-2 x^{2}+8 x-8$ does not approach a particular $y$-value as the $x$-values get larger and larger in the positive direction (or smaller and smaller in the negative direction); therefore $g(x)$ has no horizontal asymptote
- $g(x)=-2 x^{2}+8 x-8$ does not get larger and larger (or smaller and smaller) as the $x$-values approach a particular number; therefore $g(x)$ has no vertical asymptote

Table and Graph:

- Patterns in table and graph indicate $h(x)$ has no vertical or horizontal asymptotes


## Symbolically:

- $h(x)=\sqrt{x-2}+3$ does not approach a particular $y$-value as the $x$-values get larger and larger in the positive direction (or smaller and smaller in the negative direction); therefore $g(x)$ has no horizontal asymptote
- $h(x)=\sqrt{x-2}+3$ does not get larger and larger (or smaller and smaller) as the $x$-values approach a particular number; therefore $h(x)$ has no vertical asymptote


## Domain

Table and Graph:

- $\{-1,0,1,2,3,4,5,6\}$

Graphically and Symbolically:

- All real numbers
$x \in \mathfrak{R}$
$\{x \mid x \in \mathfrak{R}\}$
$(-\infty, \infty)$
Table and Graph:
- $\{4,2,0,-2,-4\}$

Graphically and Symbolically:

- All real numbers greater than or equal to -4
$y \geq-4$
$\{y \mid y \in \mathfrak{R}, y \geq-4\}$ $[-4, \infty)$

Table and Graph:

- $\{-2,-1,0,1,2,3,4\}$

Graphically and Symbolically:

- All real numbers
$x \in \Re$
$\{x \mid x \in \mathfrak{R}\}$
$(-\infty, \infty)$

Table and Graph:

- $\{-1,0,1,1.9,1.999,2,2.001,3,6,11\}$

Graphically and Symbolically:

- All real numbers greater than or equal to 2
$x \geq 2$
$\{x \mid x \in \mathfrak{R}, x \geq 2\}$
$[2, \infty)$
Table and Graph: Range
- $\{-32,-18,-8,-2,0\}$

Graphically and Symbolically:

- All real numbers less than or equal to 0

$$
\begin{aligned}
& y \leq 0 \\
& \{y \mid y \in \Re, y \leq 0\}
\end{aligned}
$$

Table and Graph:

- $\{3,3.032,4,5,6\}$

Graphically and Symbolically:

- All real numbers greater than or equal to 3
$\mathrm{y} \geq 3$
$\{y \mid y \in \mathfrak{R}, y \geq 3\}$

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## Algebraic Reasoning

Summary of key attributes of $f(x)=2|x-3|-4, g(x)=-2 x^{2}+8 x-8$, and $h(x)=\sqrt{x-2}+3$

| Attribute | $f(x)=2\|x-3\|-4$ | $g(x)=-2 x^{2}+8 x-8$ | $h(x)=\sqrt{x-2}+3$ |
| :---: | :---: | :---: | :---: |
| $x$-intercept(s) | $(1,0)$ and (5, 0) | $(2,0)$ | none |
| $y$-intercept | (0, 2) | $(0,-8)$ | none |
| Maximum | none | $(2,0)$ | none |
| Minimum | (2, 3) | none | $(2,3)$ |
| Horizontal asymptote | none | none | none |
| Vertical asymptote | none | none | none |
| Domain | All real numbers $\begin{gathered} x \in \mathfrak{R} \\ \{x \mid x \in \mathfrak{R}\} \\ (-\infty, \infty) \end{gathered}$ | $\begin{gathered} \text { All real numbers } \\ x \in \mathfrak{R} \\ \{x \mid x \in \mathfrak{R}\} \\ (-\infty, \infty) \end{gathered}$ | All real numbers greater than or equal to 2 $x \geq 2$ $\{x \mid x \in \mathfrak{R}, x \geq 2\}$ $[2, \infty)$ |
| Range | All real numbers greater than or equal to -4 $y \geq-4$ $\{y \mid y \in \mathfrak{R}, y \geq-4\}$ $[-4, \infty)$ | All real numbers less than or equal to 0 $y \leq 0$ $\begin{gathered} \{y \mid y \in \mathfrak{R}, y \leq 0\} \\ (-\infty, 0] \end{gathered}$ | All real numbers greater than or equal to 3 $y \geq 3$ $\{y \mid y \in \mathfrak{R}, y \geq 3\}$ <br> $[3, \infty)$ |

- Visual summary of relationships among the domains of linear, quadratic, square root, rational, cubic, cube root, exponential, absolute value, and logarithmic (where the base is 10 or e) functions
- Ex:

| Domain is all <br> real numbers | Domain is all real numbers <br> excluding a vertical asymptote | Domain is all real numbers on one <br> side of endpoint or vertical asymptote |
| :---: | :---: | :---: |
| Linear | Rational | Square Root <br> Absolute Value <br> Quadratic <br> Exponential <br> Cubic <br> Cube Root |
|  |  |  |

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## Algebraic REASONING

- Visual summary of relationships among the ranges of linear, quadratic, square root, rational, cubic, cube root, exponential, absolute value, and logarithmic (where the base is 10 or e) functions
- Ex:

- Visual summary of relationships among the $x$-intercepts of linear, quadratic, square root, rational, cubic, cube root, exponential, absolute value, and logarithmic (where the base is 10 or e) functions



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## Algebraic Reasoning

- Visual summary of relationships among the $y$-intercepts of linear, quadratic, square root, rational, cubic, cube root, exponential, absolute value, and logarithmic (where the base is 10 or e) functions
- Ex:

- Visual summary of relationships among the maximum and minimum values of linear, quadratic, square root, rational, cubic, cube root, exponential, absolute value, and logarithmic (where the base is 10 or e) functions
- Ex:



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- Visual summary of relationships among the asymptotes of linear, quadratic, square root, rational, cubic, cube root, exponential, absolute value, and logarithmic (where the base is 10 or e) functions
- Ex:


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Algebraic Reasoning


Note(s):

- Grade Level(s):
- Algebra I identified the key attributes of linear, quadratic, and exponential functions.
- Algebraic Reasoning introduces the key attributes of absolute value, cubic, rational, square root, cube root, and logarithmic functions with bases of 10 or $e$.
- Algebra Il will introduce and expand on the key attributes of all previous and higher degree polynomial functions.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- II.D. Algebraic Reasoning - Representing relationships
- II.D.1. Interpret multiple representations of equations, inequalities, and relationships.
- VI.A. Functions - Recognition and representation of functions
- VI.A.2. Recognize and distinguish between different types of functions.
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VIII.A.2. Use mathematical language to represent and communicate the mathematical concepts in a problem.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.


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## ALgEbRAIC REASONING

## AR.3B $\quad$ Compare and contrast the key attributes of a function and its inverse when it exists, including domain, range, maxima, minima, and

 intercepts, tabularly, graphically, and symbolically.Compare and Contrast
THE KEY ATTRIBUTES OF A FUNCTION AND ITS INVERSE WHEN IT EXISTS, INCLUDING DOMAIN, RANGE, MAXIMA, MINIMA, AND INTERCEPTS, TABULARLY, GRAPHICALLY, AND SYMBOLICALLY

Including, but not limited to:

- Function - a relation in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Key attributes of functions
- Domain and range
- Domain - a set of input values for the independent variable over which the function is defined
- Restricted domain - a set of limited domain values that allows a non-functional relation to become functional
- Range - a set of output values for the dependent variable over which the function is defined
- Notation for domain and range
- $\in$ represents an element of a set
- $\Re$ represents the set of real numbers
- Z represents the set of integers
- Q represents the set of rational numbers
- W represents the set of whole numbers
- N represent the set of natural numbers
- Representations of domain and range
- Verbal description
- Interval notation - notation in which the solution is represented by a continuous interval
- Parentheses indicate that the endpoints are open, meaning the endpoints are excluded from the interval.
- Negative infinity, $-\infty$, and positive infinity, $\infty$, are always associated with a parenthesis.
- Brackets indicate that the endpoints are closed, meaning the endpoints are included in the interval.
- Inequality notation - notation in which the solution is represented by an inequality statement
- Set (builder) notation - notation in which the solution is represented by a set of values
- Braces are used to enclose the set.
- $\{x \mid x \in$ is read as "The set of $x$ such that $x$ is an element of ..."
- A set could be a list of values contained within braces; e.g., $\{1,2,3,4,5\}$.
- Ex:


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Algebraic Reasoning

|  | - Asymptote - a line that is approached and may or may not be crossed <br> - Horizontal asymptote - horizontal line approached by the curve as the function approaches positive or negative infinity. Horizontal asymptotes may be crossed by the curve. <br> - Exponential and rational functions have horizontal asymptotes <br> - Vertical asymptote - vertical line approached by the curve as the function approaches positive or negative infinity. Vertical asymptotes are never crossed by the curve. <br> - Logarithmic and rational functions have vertical asymptotes <br> - Asymptotic behavior - the graph of a function such that as $x$ approaches positive or negative infinity (a particular value), $f(x)$ or $y$ approaches a given value (negative or positive infinity) <br> - Maximum - largest $y$-coordinate or $y$-value a function takes over the entire domain of the curve represented as a coordinate set <br> - Minimum - smallest $y$-coordinate or $y$-value a function takes over the entire domain of the curve represented as a coordinate set <br> - $x$-intercept $-x$-coordinate of a point at which the relation crosses the $x$-axis, meaning the $y$-coordinate equals zero, $(x, 0)$ <br> - Zeros - the value(s) of $x$ such that the $y$-value of the relation equals zero <br> - Functions comprised of polynomials may have up to the degree number of $x$-intercepts <br> - Exponential, logarithmic, rational, cube root, and square root functions have at most one $x$-intercept <br> - Absolute value functions involving the absolute value of a linear expression may have zero, one, or two $x$-intercepts <br> - $y$-intercept - $y$-coordinate of a point at which the relation crosses the $y$-axis, meaning the $x$-coordinate equals zero, $(0, y)$ <br> - Functions with a domain of all real numbers have one $y$-intercept. <br> - Functions where the dependent value is undefined when the independent value is 0 will not have a $y$-intercept. <br> - Relationships between functions and their inverses <br> - All inverses of functions are relations. <br> - Reflections across the line $y=x$ and symmetrical about the line $y=x$ <br> - Domain of function is range of inverse relation and range of function is domain of inverse relation |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |

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## Algebraic Reasoning

- Corresponding points between function and inverse relation have interchanged $x$ - and $y$-coordinate values
- Graph of function including $(3.5,-7)$ implies graph of inverse relation includes $(-7,3.5)$
- Functionality of inverse relation of the given function
- One-to-one function - function where every element of the range corresponds to exactly one element of the domain
- Inverse relation of a one-to-one function is a function
- Linear function and linear function
- Ex:

Given tabular, graphical, and symbolic representations for $f(x)=-\frac{1}{2} x-4$ and its inverse, $g(x)=-2 x-8$, compare and contrast the key attributes of $f(x)$ and $g(x)$.

Tabular

$$
f(x)=-\frac{1}{2} x-4 \quad g(x)=-2 x-8
$$



Graphical


## Reflection across $y=x$

The function $f(x)=-\frac{1}{2} x-4$ and its inverse $g(x)=-2 x-8$ have interchanged $x$ - and $y$-coordinate values. For example, $f(-8)=8$ and $g(8)=-8$. The graph of $f(x)$ and its inverse $g(x)$ are reflections across the line, $y=x$, modeling the interchanged $x$ - and $y$-coordinate value relationship.

Domain and Range

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## Algebraic Reasoning

The function $f(x)=-\frac{1}{2} x-4$ and its inverse $g(x)=-2 x-8$ have an interchanged domain and range. For example, tabularly, the domain of $f(x)$ is $\{-14,-8,-6,0,2,8\}$, which is the range of $g(x)$; and the range of $f(x)$ is $\{3,0,-1,-4,-5,-8\}$, which is the domain of $g(x)$. Graphically, the domain of $f(x)$ is all real numbers, which is the range of $g(x)$. Likewise, the range of $f(x)$ is all real numbers, which is the domain of $g(x)$.

## $x$-intercept(s) and $y$-intercept

The $x$-intercept of $f(x)$ is $(-8,0)$ and the $y$-intercept of $g(x)$ is $(0,-8)$. Likewise, the $y$-intercept of $f(x)$ is $(0,-4)$ and the $x$-intercept of $g(x)$ is $(-4,0)$. The interchanging of $x$ - and $y$-coordinate values between a function and its inverse results in interchanged $x$ - and $y$-coordinate intercepts.

> Maximum and Minimum

Since the functions $f(x)$ and $g(x)$ are linear functions, there are no maximum or minimum values.
Asymptote(s)

Since the functions $f(x)$ and $g(x)$ are linear functions, there are no horizontal or vertical asymptotes.
Summary of key attributes of $f(x)=-\frac{1}{2} x-4$ and its inverse $g(x)=-2 x-8$


- Cubic function and cube root function
- Ex:


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## Algebraic ReAsoning

Given tabular, graphical, and symbolic representations for $f(x)=\frac{1}{2}(x+2)^{3}+4$ and its inverse, $g(x)=\sqrt[3]{2(x-4)}-2$, compare and contrast the key attributes of $f(x)$ and $g(x)$.

$$
\begin{aligned}
& \text { Tabular } \\
& f(x)=\frac{1}{2}(x+2)^{3}+4 \quad g(x)=\sqrt[3]{2(x-4)}-2
\end{aligned}
$$

| $\boldsymbol{x}$ | $f(x)$ | $\underset{y \text {-int } \rightarrow}{\leftarrow x \text { int }}$ | $\boldsymbol{x}$ | $g(x)$ |
| :---: | :---: | :---: | :---: | :---: |
| -6 | -28 |  | -28 | -6 |
| -4 | 0 |  | 0 | -4 |
| -2 | 4 | $\underset{x \text {-int } \rightarrow}{\leftarrow-\text { int }}$ | 4 | -2 |
| 0 | 8 |  | 8 | 0 |
| 1 | 17.5 |  | 17.5 | 1 |
| 2 | 36 |  | 36 | 2 |

Reflection across $y=x$
The function $f(x)=\frac{1}{2}(x+2)^{3}+4$ and its inverse $g(x)=\sqrt[3]{2(x-4)}-2$ have interchanged $x$ - and $y$-coordinate values. For example, $f(-2)=4$ and $g(4)=-2$. The graph of $f(x)$ and its inverse $g(x)$ are reflections across the line, $y=x$, modeling the interchanged $x$ - and $y$-coordinate value relationship.
Domain and Range
The function $f(x)=\frac{1}{2}(x+2)^{3}+4$ and its inverse $g(x)=\sqrt[3]{2(x-4)}-2$ have an interchanged domain and range. For example, tabularly, the domain of $f(x)$ is $\{-6,-4,-2,0,1,2\}$, which is the range of $g(x)$; and the range of $f(x)$ is $\{-28,0,4,8,17.5,36\}$, which is the domain of $g(x)$. Graphically, the domain of $f(x)$ is all real numbers, which is the range of $g(x)$. Likewise, the range of $f(x)$ is all real numbers, which is the domain of $g(x)$,

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## Algebraic Reasoning

The $x$-intercept of $f(x)$ is $(-4,0)$ and the $y$-intercept of $g(x)$ is $(0,-4)$. Likewise, the $y$-intercept of $f(x)$ is $(0,8)$ and the $x$-intercept of $g(x)$ is $(8,0)$. The interchanging of $x$ - and $y$-coordinate values between a function and its inverse results in interchanged $x$ - and $y$-intercepts.

Maximum and Minimum
Since $f(x)$ is a cubic function and $g(x)$ is a cube root function, there are no maximum or minimum values.
Asymptote(s)
Since the functions $f(x)$ and $g(x)$ are cubic functions, there are no horizontal or vertical asymptotes.
Summary of key attributes of $f(x)=\frac{1}{2}(x+2)^{3}+4$ and its inverse $g(x)=\sqrt[3]{2(x-4)}-2$

| Attribute | $f(x)=\frac{1}{2}(x+2)^{3}+4$ | $g(x)=\sqrt[3]{2(x-4)}-2$ |
| :---: | :---: | :---: |
| Domain | All real numbers | All real numbers |
|  | $x \in \mathfrak{R}$ | $x \in \mathfrak{R}$ |
|  | $\{x \mid x \in \mathfrak{R}\}$ | $\{x \mid x \in \mathfrak{R}\}$ |
|  | $(-\infty, \infty)$ | $(-\infty, \infty)$ |
| Range | All real numbers | All real numbers |
|  | $y \in \mathfrak{R}$ | $y \in \mathfrak{R}$ |
|  | $\{y \mid y \in \Re\}$ | $\{y \mid y \in \mathfrak{R}\}$ |
|  | $(-\infty, \infty)$ | $(-\infty, \infty)$ |
| $x$-intercept | $(-4,0)$ | $(8,0)$ |
| $y$-intercept | $(0,8)$ | $(0,-4)$ |
| Maximum | none | none |
| Minimum | none | none |
| Asymptotes | none | none |
|  |  |  |

- Exponential function and logarithmic function
- Ex:

Given tabular, graphical, and symbolic representations for $f(x)=e^{x}+2$ and its inverse, $g(x)=\ln (x-2)$ or $g(x)=\log _{e}(x-2)$, compare and contrast the key attributes of $f(x)$ and $g(x)$.
(e approximated as 2.718;
values in table rounded to hundredths)

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## Algebraic REASONING



The function $f(x)=e^{x}+2$ and its inverse $g(x)=\ln (x-2)$ or $g(x)=\log _{e}(x-2)$ have interchanged $x$ - and $y$-coordinate values. For example, $f(2)=9.39$ and $g(9.39)=2$. The graph of $f(x)$ and its inverse $g(x)$ are reflections across the line, $y=x$, modeling the interchanged $x$ - and $y$-coordinate value relationship.

> Domain and Range

The function $f(x)=e^{x}+2$ and its inverse $g(x)=\ln (x-2)$ or $g(x)=\log _{e}(x-2)$ have an interchanged domain and range. For example, tabularly, the domain of $f(x)$ is $\{-5,-3,-2,-1,0,1,2\}$, which is the range of $g(x)$; and the range of $f(x)$ is $\{2.01,2.05,2.14,2.37,3,4.72,9.39\}$, which is the domain of $g(x)$. Graphically, the domain of $f(x)$ is all real numbers, which is the range of $g(x)$. Likewise, the range of $f(x)$ is all real numbers greater than 2 , which is the domain of $g(x)$.

$$
x \text {-intercept(s) and } y \text {-intercept }
$$

There is no $x$-value such that $f(x)=0$, so $f(x)$ has no $x$-intercept, and there is no $y$-value such that $f(0)$ is defined, so $g(x)$ has no $y$-intercept. The $y$-intercept of $f(x)$ is $(0,3)$ and the $x$-intercept of $g(x)$ is $(3,0)$. The interchanging of $x$ - and $y$-coordinate values between a function and its inverse results in interchanged $x$ - and $y$-coordinate intercepts.

## Maximum and Minimum

Since the function $f(x)$ is an exponential function with a horizontal asymptote and $g(x)$ is a logarithmic function with a vertical asymptote, there are no maximum or minimum values.

## Asymptote(s)

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## Algebraic Reasoning

The function $f(x)$ has a horizontal asymptote at the line $y=2$ and $g(x)$ has a vertical asymptote at the line $x=2$. The interchanging of the line value of 2 for the asymptotes of $f(x)$ and $g(x)$ demonstrates that $f(x)$ and $g(x)$ are inverse functions. The interchanging of $x$ - and $y$-coordinate values between a function and its inverse results in interchanged asymptotes.

| Summary of key attributes of $f(x)=e^{x}+2$ and its inverse $g(x)=\ln (x-2)$ |  |  |
| :---: | :---: | :---: |
| Attribute | $f(x)=e^{x}+2$ | $g(x)=\ln (x-2)$ |
| Domain | All real numbers $\begin{gathered} x \in \mathfrak{R} \\ \{x \mid x \in \mathfrak{R}\} \\ (-\infty, \infty) \end{gathered}$ | All real numbers greater than 2 $x>2$ $\{x \mid x \in \mathfrak{R}, x>2\}$ $(2, \infty)$ |
| Range | All real numbers greater than 2 $y>2$ $\{y \mid y \in \Re, y>2\}$ <br> $(2, \infty)$ | All real numbers $\begin{gathered} y \in \mathfrak{R} \\ \{y \mid y \in \mathfrak{R}\} \\ (-\infty, \infty) \end{gathered}$ |
| $x$-intercept | none | $(3,0)$ |
| $y$-intercept | $(0,3)$ | none |
| Maximum | none | none |
| Minimum | none | none |
| Asymptotes | $y=2$ | $x=2$ |

- Inverse of a function that is not one-to-one can be made a function by restricting the domain of the original function
- Quadratic function and positive square root function
- Ex:

Given tabular, graphical, and symbolic representations for $f(x)=x^{2}-4$ and its inverse, $g(x)=+\sqrt{x+4}$, compare and contrast the key attributes of $f(x)$ and $g(x)$.

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The function $f(x)=x^{2}-4$ and its inverse $g(x)=-\sqrt{x+4}$ have interchanged $x$ - and $y$-coordinate values. For example, $f(-3)=5$ and $g(5)=-3$. The graph of $f(x)$ with its restricted domain and its inverse $g(x)$ are reflections across the line, $y=x$, modeling the interchanged $x$ - and $y$-coordinate value relationship.

The function $f(x)=x^{2}-4$ and its inverse $g(x)=-\sqrt{x+4}$ have an interchanged domain and range when $f(x)$ is restricted to domain values $x \leq 0$. For example, tabularly, the restricted domain of $f(x)$ is $\{-3,-2,-1,0\}$, which is the range of $g(x)$; and the range of $f(x)$ is $\{5,0,-3,-4\}$, which is the domain of $g(x)$. Graphically, the restricted domain of $f(x)$ is all real numbers less than or equal to 0 , which is the range of $g(x)$. Likewise, the range of $f(x)$ is all real numbers greater than or equal to -4 , which is the domain of $g(x)$.
$x$-intercept(s) and $y$-intercept
The $x$-intercept of $f(x)$ with its restricted domain is $(-2,0)$ and the $y$-intercept of $g(x)$ is $(0,-2)$. Likewise, the $y$-intercept of $f(x)$ is $(0,-4)$ and the $x$-intercept of $g(x)$ is $(-4,0)$. The interchanging of $x$ - and $y$-coordinate values between a function and its inverse results in interchanged $x$ - and $y$-coordinate intercepts.

Maximum and Minimum

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## Algebraic Reasoning

The minimum of $f(x)$ with its restricted domain is -4 , located at $(0,-4)$, and the maximum of $g(x)$ is 0 , located at $(-4,0)$. The interchanging of $x$ - and $y$-coordinate values between a function and its inverse results in interchanged minimum and maximum values.
Asymptote
Since the function $f(x)$ with its restricted domain is quadratic and $g(x)$ is a negative square root function, there are no horizontal or
vertical asymptotes.

Summary of key attributes of $f(x)=x^{2}-4$ and its inverse $g(x)=-\sqrt{x+4}$

| Attribute | $f(x)=x^{2}-4$ | $g(x)=-\sqrt{x+4}$ |
| :---: | :---: | :---: |
| Domain | All real numbers less than or equal to 0 $x \leq 0$ $\{x \mid x \in \Re, x \leq 0\}$ $(-\infty, 0]$ | All real numbers greater than or equal to -4 $\begin{gathered} x \geq-4 \\ \{x \mid x \in \mathfrak{R}, x \geq-4\} \end{gathered}$ $[-4, \infty)$ |
| Range | All real numbers greater than or equal to -4 $\begin{gathered} y \geq-4 \\ \{y \mid y \in \mathfrak{R}, y \geq-4\} \\ {[-4, \infty)} \end{gathered}$ | All real numbers less than or equal to 0 $\begin{gathered} y \leq 0 \\ \{y \mid y \in \mathfrak{R}, y \leq 0\} \\ (-\infty, 0] \end{gathered}$ |
| $x$-intercept | $\square(-2,0)$ | $(-4,0)$ |
| $y$-intercept | $(0,-4)$ | $(0,-2)$ |
| Maximum | none | $(-4,0)$ |
| Minimum | (0, -4) | none |
| Asymptotes | $\rightarrow$ none | none |

- Absolute value function and positive linear function
- Ex:

Given tabular, graphical, and symbolic representations for $f(x)=|x+3|+2$ and its inverse, $g(x)=x-5$, for $x \geq 2$, compare and contrast the key attributes of $f(x)$ and $g(x)$.

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## Algebraic REASONING

Tabular
$f(x)=|x+3|+2 \quad g(x)=x-5$, for $x \geq 2$

| $\boldsymbol{x}$ | $f(x)$ |
| :---: | :---: |
| -9 | 8 |
| -6 | 5 |
| -4 | 3 |
| -3 | 2 |
| -2 | 3 |
| 0 | 5 |
| 3 | 8 |


| $x$ | $g(x)$ |
| :---: | :---: |
| 8 | 3 |
| 5 | 0 |
| 3 | -2 |
| 2 | -3 |
| 3 | -2 |
| 5 | 0 |
| 8 | 3 |

## Graphical

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## Algebraic Reasoning

Since the function $f(x)$ with its restricted domain is absolute value and $g(x)$ is a linear function, there are no horizontal or vertical asymptotes.

Summary of key attributes of $f(x)=|x+3|+2$ and its inverse $g(x)=x-5$, for $x \geq 2$

| Attribute | $f(x)=\|x+3\|+2$ | $g(x)=x-5$, for $x \geq 2$ |
| :---: | :---: | :---: |
| Domain | All real numbers greater than or equal to -3 $\begin{gathered} x \geq-3 \\ \{x \mid x \in \mathfrak{R}, x \geq-3\} \\ {[-3, \infty)} \\ \hline \end{gathered}$ | All real numbers greater than or equal to 2 $\begin{gathered} x \geq 2 \\ \{x \mid x \in \mathfrak{R}, x \geq 2\} \\ {[2, \infty)} \\ \hline \end{gathered}$ |
| Range | All real numbers greater than or equal to 2 $y \geq 2$ <br> $\{y \mid y \in \mathfrak{R}, y \geq 2\}$ <br> $[2, \infty)$ | All real numbers greater than or equal to -3 $\begin{gathered} y \geq-3 \\ \{y \mid y \in \mathfrak{R}, y \geq-3\} \\ {[-3, \infty)} \\ \hline \end{gathered}$ |
| $x$-intercept | none | $(5,0)$ |
| $y$-intercept | (0,5) | none |
| Maximum | none | none |
| Minimum | $(-3,2)$ | (2, -3) |
| Asymptotes | none | none |

- Absolute value function and negative linear function
- Ex:

> Given tabular, graphical, and symbolic representations for $f(x)=|x+3|+2$ and its inverse, $g(x)=-x-1$, for $x \geq 2$, compare and contrast the key attributes of $f(x)$ and $g(x)$.

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## Algebraic Reasoning

## Asymptote

Since the function $f(x)$ with its restricted domain is absolute value and $g(x)$ is a linear function, there are no horizontal or vertical asymptotes.

Summary of key attributes of $f(x)=|x+3|+2$ and its inverse $g(x)=-x-1$, for $x \geq 2$

| Attribute | $f(x)=\|x+3\|+2$ | $g(x)=-x-1$, for $x \geq 2$ |
| :---: | :---: | :---: |
| Domain | $\begin{gathered} \hline \text { All real numbers less than } \\ \text { or equal to }-3 \\ x \leq-3 \\ \{x \mid x \in \Re, x \leq-3\} \\ (-\infty,-3] \\ \hline \end{gathered}$ | All real numbers greater than or equal to 2 $\begin{gathered} x \geq 2 \\ \{x \mid x \in \mathfrak{R}, x \geq 2\} \\ {[2, \infty)} \\ \hline \end{gathered}$ |
| Range | All real numbers greater than or equal to 2 $y \geq 2$ $\{y \mid y \in \Re, y \geq 2\}$ <br> $[2, \infty)$ | $\begin{gathered} \text { All real numbers less than } \\ \text { or equal to }-3 \\ y \leq-3 \\ \{y \mid y \in \mathfrak{R}, y \leq-3\} \\ (-\infty,-3] \end{gathered}$ |
| $x$-intercept | none | none |
| $y$-intercept | none | none |
| Maximum | none | $(2,-3)$ |
| Minimum | $\square(-3,2)$ | none |
| Asymptotes | none | none |

Note(s):

- Grade Level(s):
- Algebra 1 determined if relations represent a function.
- Algebraic Reasoning introduces the idea of the inverse of a function and its relationship tabularly, graphically, and symbolically.
- Algebraic Reasoning introduces the comparison of attributes of a function and its inverse and the verification of whether or not two functions are inverses, including domain and range restrictions as necessary.
- Algebraic Reasoning does not require students to generate the inverse of a function symbolically.
- Algebra II will introduce using composition to verify whether or not two functions are inverses.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- II.D. Algebraic Reasoning - Representing relationships
- II.D.1. Interpret multiple representations of equations, inequalities, and relationships.
- VI.A. Functions - Recognition and representation of functions


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## Algebraic ReAsoning

- VI.A.2. Recognize and distinguish between different types of functions.
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.2. Use mathematical language to represent and communicate the mathematical concepts in a problem.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications

Verify that two functions are inverses of each other tabularly and graphically such as situations involving compound interest and interest rate, velocity and braking distance, and Fahrenheit-Celsius conversions.

## Verify

THAT TWO FUNCTIONS ARE INVERSES OF EACH OTHER TABULARLY AND GRAPHICALLY SUCH AS SITUATIONS INVOLVING COMPOUND INTEREST AND INTEREST RATE, VELOCITY AND BRAKING DISTANCE, AND FAHRENHEIT-CELSIUS CONVERSIONS

Including, but not limited to:

- Function - a relation in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Inverse of a function - function that undoes the original function
- Relationships between functions and their inverses
- All inverses of functions are relations.
- Reflections across the line $y=x$ and symmetrical about the line $y=x$
- Corresponding points between function and inverse relation have interchanged $x$ - and $y$-coordinate values
- Graph of function including $(3.5,-7)$ implies graph of inverse relation includes $(-7,3.5)$
- Domain of function is range of inverse relation and range of function is domain of inverse relation
- Ex:

Given tabular and graphical representations for $f(x)=2 x+3$ and $g(x)=\frac{x-3}{2}$, verify whether or not $f(x)$ and $g(x)$ are inverses.
$f(x)=2 x+3$ $x-3$

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- Ex:

Given tabular and graphical representations for $m(x)=(x+3)^{2}$ and $n(x)=-(x-3)^{2}$, verify whether or not $m(x)$ and $n(x)$ are inverses

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## Algebraic REASONING

Given tabular and graphical representations for the compound interest formula, $A=P(1+r)^{t}$ and the interest rate formula, $r=\left(\frac{A}{P}\right)^{\frac{1}{t}}-1$, verify the functions are inverses when the initial principal, $P$, is $\$ 10$ and the loan is for 60 pay periods, $t$.

Tabular

$$
A(r)=10(1+r)^{60}
$$

$$
r(A)=\left(\frac{A}{10}\right)^{\frac{1}{60}}-1
$$

| $\boldsymbol{r}$ | $\boldsymbol{A}(\boldsymbol{r})$ |
| :---: | :---: |
| 0.00 | 10.00 |
| 0.01 | 18.17 |
| 0.02 | 32.81 |
| 0.03 | 58.92 |
| 0.04 | 105.20 |
| 0.05 | 186.79 |


| $\boldsymbol{A}$ | $\boldsymbol{r}(\boldsymbol{A})$ |
| :---: | :---: |
| 10.00 | 0.00 |
| 18.17 | 0.01 |
| 32.81 | 0.02 |
| 58.92 | 0.03 |
| 105.20 | 0.04 |
| 186.79 | 0.05 |

The table shows the functions $A(r)=10(1+r)^{60}$ and $r(A)=\left(\frac{A}{10}\right)^{\frac{1}{60}}-1$ have interchanged $x$ - and $y$-coordinate values. For example, $A(0.02)=32.81$ and $r(32.81)=0.02$. The domain of $A(r)$ is $\{0,0.01,0.02,0.03,0.04,0.05\}$, which is the range of $r(A)$; and the range of $A(r)$ is $\{10,18.17,32.81,58.92,105.20,186.79\}$, which is the domain of $r(A)$. Since $A(r)$ and $r(A)$ have interchanged $x$ - and $y$-coordinate values, $A(r)$ and $r(A)$ are inverses of each other.


The graph shows interchanged $x$ - and $y$-coordinate values for $A(r)$ and $r(A)$. For example, $(0.03,58.92)$ and (58.92, 0.03) have interchanged coordinate values. Points with interchanged coordinate values are reflections across the line $y=x$; therefore, $A(r)$ and $r(A)$ are reflections across the line $y=x$. Functions reflected across $y=x$ are inverses; therefore, $A(r)$ and $r(A)$ are inverses of each other.

- Velocity
- $v=\sqrt{17.64 d}$, where $v$ represents velocity and $d$ represents the distance over a set time


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## Algebraic Reasoning

- Braking distance
- $d=\frac{v^{2}}{17.64}$, where $d$ represents the distance over a set time and $v$ represents velocity
- Ex:

Given tabular and graphical representations of the velocity formula, $v=\sqrt{17.64 d}$, and the braking distance formula, $d=\frac{v^{2}}{17.64}$, verify the functions are inverses of each other.

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## Algebraic Reasoning

Tabular
$v(d)=\sqrt{17.64 d}$

| $\boldsymbol{d}$ | $\mathbf{v}(\boldsymbol{d})$ |
| :---: | :---: |
| 0 | 0 |
| 50 | 29.70 |
| 100 | 42.00 |
| 150 | 51.44 |
| 200 | 59.40 |
| 300 | 72.75 |

$$
d(v)=\frac{v^{2}}{17.64}
$$

| $v$ | $d(v)$ |
| :---: | :---: |
| 0 | 0 |
| 29.70 | 50 |
| 42.00 | 100 |
| 51.44 | 150 |
| 59.40 | 200 |
| 72.75 | 300 |

The table shows the functions $v(d)=\sqrt{17.64 d}$ and $d(v)=\frac{v^{2}}{17.64}$ have interchanged $x$ - and $y$-coordinate values. For example, $v(100)=42$ and $d(42)=100$. The domain of $v(d)$ is $\{0,50,100,150,200,300\}$, which is the range of $d(v)$; and the range of $v(d)$ is
$\{0,29.7,42,51.44,59.4,72.75\}$, which is the domain of $d(v)$. Since $v(d)$ and $d(v)$ have interchanged $x$ - and $y$-coordinate values, $v(d)$ and $d(v)$ are inverses of each other.

Graphical


The graph shows interchanged $x$ - and $y$-coordinate values for $v(d)$ and $d(v)$. For example, $(200,59.4)$ and
$(59.4,200)$ have interchanged coordinate values. Points with interchanged coordinate values are reflections across the line $y=x$; therefore, $v(d)$ and $d(v)$ are reflections across the line $y=x$. Functions reflected across $y=x$ are inverses; therefore, $v(d)$ and $d(v)$ are inverses of each other.

- Fahrenheit to Celsius
- $C(F)=\frac{5}{9}(F-32)$, where $C$ represents temperature measured in Celsius and $F$ represents temperature measured in Fahrenheit
- Celsius to Fahrenheit
- $F(C)=\frac{9}{5} C+32$, where $F$ represents temperature measured in Fahrenheit and $C$ represents temperature measured in Celsius


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## Algebraic REASONING

- Ex:

Given tabular and graphical representations of the Fahrenheit to Celsius formula, $C(F)=\frac{5}{9}(F-32)$, and the Celsius to Fahrenheit formula, $F(C)=\frac{9}{5} C+32$, verify the functions are inverses of each other.

$$
C(F)=\frac{5}{9}(F-32) \quad F(C)=\frac{9}{5} C+32
$$

| $\boldsymbol{F}$ | $\boldsymbol{C}(\boldsymbol{F})$ |
| :---: | :---: |
| -13 | -25 |
| 5 | -15 |
| 14 | -10 |
| 32 | 0 |
| 41 | 5 |
| 68 | 20 |


| $C$ | $F(C)$ |
| :---: | :---: |
| -25 | -13 |
| -15 | 5 |
| -10 | 14 |
| 0 | 32 |
| 5 | 41 |
| 20 | 68 |

The table shows the functions $C(F)=\frac{5}{9}(F-32)$ and $F(C)=\frac{9}{5} C+32$ have interchanged $x$ - and $y$-coordinate values. For example, $C(32)=0$ and $F(0)=32$. The domain of $C(F)$ is $\{-13,5,14,32,41,68\}$, which is the range of $F(C)$; and the range of $C(F)$ is $\{-25,-15,-10,0,5,20\}$, which is the domain of $F(C)$. Since $C(F)$ and $F(C)$ have interchanged $x$ - and $y$-coordinate values, $C(F)$ and $F(C)$ are inverses of each other.


The graph shows interchanged $x$ - and $y$-coordinate values for $C(F)$ and $F(C)$. For example, $(5,-15)$ and $(-15,5)$ have interchanged coordinate values. Points with interchanged coordinate values are reflections across the line $y=x$; therefore, $C(F)$ and $F(C)$ are reflections across the line $y=$ $x$. Functions reflected across $y=x$ are inverses; therefore, $C(F)$ and $F(C)$ are inverses of each other.

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Note(s):

- Grade Level(s):
- Algebra I introduced quadratic and exponential functions.
- Algebraic Reasoning introduces inverse functions and their behavior tabularly and graphically.
- Algebraic Reasoning does not require students to generate the inverse of a function symbolically.
- Algebra Il will generate inverses of functions using algebraic methods.
- Algebra Il will introduce using composition to verify whether or not two functions are inverses.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- II.D. Algebraic Reasoning - Representing relationships
- II.D.1. Interpret multiple representations of equations, inequalities, and relationships.
- VI.A. Functions - Recognition and representation of functions
- VI.A.2. Recognize and distinguish between different types of functions.
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions.
- VI.C. Functions - Model real-world situations with functions
- VI.C.1. Apply known functions to model real-world situations.
- VI.C.2. Develop a function to model a situation.
- VII.D. Problem Solving and Reasoning - Real-world problem solving
- VII.D.1. Interpret results of the mathematical problem in terms of the original real-world situation.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
- IX.A. Connections - Connections among the strands of mathematics
- IX.A.2. Connect mathematics to the study of other disciplines.
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
- IX.B.2. Understand and use appropriate mathematical models in the natural, physical, and social sciences.

Represent a resulting function tabularly, graphically, and symbolically when functions are combined or separated using arithmetic operations such as combining a $20 \%$ discount and a $6 \%$ sales tax on a sale to determine $h(x)$, the total sale, $f(x)=0.8 x, g(x)=0.06(0.8 x)$, and $h(x)=f(x)+g(x)$.

Represent

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## Algebraic Reasoning

 A RESULTING FUNCTION TABULARLY, GRAPHICALLY, AND SYMBOLICALLY WHEN FUNCTIONS ARE COMBINED OR SEPARATED USING ARITHMETIC OPERATIONS SUCH AS COMBINING A $20 \%$ DISCOUNT AND A $6 \%$ SALES TAX ON A SALE TO DETERMINE $h(x)$, THE TOTAL SALE, $f(x)=0.8 x, g(x)=0.06(0.8 x)$, AND $h(x)=f(x)+g(x)$Including, but not limited to:

- Function - a relation in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Combining functions
- Addition
- $h(x)=f(x)+g(x)$, where $f(x)$ and $g(x)$ are functions
- Dependent values of individual functions are added to determine dependent values of combined function for each particular independent value
- Ex:

$$
\text { Determine } h(x) \text { if } h(x)=f(x)+g(x), f(x)=2 x+1 \text {, and } g(x)=x-3 \text {. Represent } h(x) \text { tabularly, graphically, and symbolically. }
$$

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## Algebraic Reasoning

Symbolic
$f(x)=2 x+1$
$g(x)=x-3$
$h(x)=f(x)+g(x)$
$h(x)=(2 x+1)+(x-3)$
$h(x)=2 x+1+x-3$
$h(x)=2 x+x+1-3$
$h(x)=3 x+1-3$
$h(x)=3 x-2$
Evaluating $h(x)$ :
$h(x)=f(x)+g(x)$
$h(x)=(2 x+1)+(x-3)$
$h(3)=(2(3)+1)+((3)-3)$
$h(3)=(6+1)+(3-3)$
$h(3)=7+0$
$h(3)=7$
OR
$h(x)=3 x-2$
$h(3)=3(3)-2$
$h(3)=9-2$
$h(3)=7$

Tabularly
Generate a list of dependent values of $h(x)$ for $h(x)=f(x)+g(x), f(x)=2 x+1$, and $g(x)=x-3$, when given a list of independent values.

| $x$ | $f(x)$ <br> $2 x+1$ | $g(x)$ <br> $x-3$ | $f(x)+g(x)$ | $h(x)$ <br> $3 x-2$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | -3 | $1+(-3)$ | -2 |
| 2 | 5 | -1 | $5+(-1)$ | 4 |
| 5 | 11 | 2 | $11+2$ | 13 |
| 9 | 19 | 6 | $19+6$ | 25 |
| 13 | 27 | 10 | $27+10$ | 37 |
| 20 | 41 | 17 | $41+17$ | 58 |

## Graphical

Generate dependent values of $h(x)$ for $h(x)=f(x)+g(x), f(x)=2 x+1$, and $g(x)=x-3$, when given independent values.


The symbolic, tabular, and graphical representations show the dependent value for $h(x)$ evaluated at a particular value of $x$ is equivalent to adding the dependent values for $f(x)$ and $g(x)$ evaluated at the same value of $x$.

- Multiplication
- $h(x)=f(x) \bullet g(x)$, where $f(x)$ and $g(x)$ are functions
- Dependent values of individual functions are multiplied to determine dependent values of combined function for each particular independent value
- Ex:

Determine $h(x)$ if $h(x)=f(x) \bullet g(x)$ and $f(x)=2 x+1$ and $g(x)=x-3$. Represent $h(x)$ tabularly, graphically, and symbolically.

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## Mathematics Enhanced TEKS Clarification Document

## Algebraic Reasoning

Symbolic
$f(x)=2 x+1$
$g(x)=x-3$
$h(x)=f(x) \bullet g(x)$
$h(x)=(2 x+1)(x-3)$
$h(x)=2 x^{2}-6 x+1 x-3$
$h(x)=2 x^{2}-5 x-3$
Evaluating $h(x)$ :
$h(x)=f(x) \bullet g(x)$
$h(x)=(2 x+1) \cdot(x-3)$
$h(4)=(2(4)+1) \cdot((4)-3)$
$h(4)=(8+1) \cdot(4-3)$
$h(4)=(9) \bullet 1$
$h(4)=9$
OR
$h(x)=2 x^{2}-5 x-3$
$h(4)=2(4)^{2}-5(4)-3$
$h(4)=2(16)-20-3$
$h(4)=32-23$
$h(4)=9$

Tabular
Generate a list of dependent values of $h(x)$ for $h(x)=f(x) \bullet g(x), f(x)=2 x+1$, and $g(x)=x-3$, when given a list of independent values.

| $x$ | $f(x)$ <br> $2 x+1$ | $g(x)$ <br> $x-3$ | $f(x) \cdot g(x)$ | $h(x)$ <br> $2 x^{2}-5 x-3$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | -3 | $1 \bullet(-3)$ | -3 |
| 2 | 5 | -1 | $5 \bullet(-1)$ | -5 |
| 5 | 11 | 2 | $11 \bullet 2$ | 22 |
| 9 | 19 | 6 | $19 \bullet 6$ | 114 |
| 13 | 27 | 10 | $27 \bullet 10$ | 270 |
| 20 | 41 | 17 | $41 \bullet 17$ | 697 |

The symbolic, tabular, and graphical representations show the dependent value for $h(x)$ evaluated at a particular value of $x$ is equivalent to multiplying the dependent values for $f(x)$ and $g(x)$ evaluated at the same value of $x$.

- Ex:

Determine $h(x)$ if $h(x)=f(x) \bullet g(x), f(x)=\frac{2}{x+3}$, and $g(x)=(x+3)^{2}$. Represent $h(x)$ tabularly, graphically, and symbolically.

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## Algebraic Reasoning

$$
\begin{aligned}
& h(x)=2 x+6 \\
& h(0)=2(0)+6 \\
& h(0)=0+6 \\
& h(0)=6
\end{aligned}
$$

The symbolic, tabular, and graphical representations show the dependent value for $h(x)$ evaluated at a particular value of $x$ is equivalent to multiplying the dependent values for $f(x)$ and $g(x)$ evaluated at the same value of $x$.

- Separating functions
- Subtraction
- $h(x)=f(x)-g(x)$, where $f(x)$ and $g(x)$ are functions
- Dependent values of individual functions are subtracted to determine dependent values of separated function for each particular independent value
- Ex:


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## Algebraic Reasoning

$h(x)=x+4$
$h(2)=(2)+4$
$h(2)=2+4$
$h(2)=6$

| $x$ | $f(x)$ <br> $2 x+1$ | $g(x)$ <br> $x-3$ | $f(x)-g(x)$ | $h(x)$ <br> $x+4$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | -3 | $1-(-3)$ | 4 |
| 2 | 5 | -1 | $5-(-1)$ | 6 |
| 5 | 11 | 2 | $11-2$ | 9 |
| 9 | 19 | 6 | $19-6$ | 13 |
| 13 | 27 | 10 | $27-10$ | 17 |
| 20 | 41 | 17 | $41-17$ | 24 |



The symbolic, tabular, and graphical representations show the dependent value for $h(x)$ evaluated at a particular value of $x$ is equivalent to subtracting the dependent values for $f(x)$ and $g(x)$ evaluated at the same value of $x$.

- Division
- $h(x)=\frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are functions and $g(x) \neq 0$
- Dependent values of individual functions are divided to determine dependent values of separated function for each particular independent value
- Ex:

$$
\text { Determine } h(x) \text { if } h(x)=\frac{f(x)}{g(x)}, f(x)=2 x+1 \text {, and } g(x)=x-3 \text {. Represent } h(x) \text { tabularly, graphically, and symbolically. }
$$

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## Algebraic Reasoning

The formula for the area of the composite figure is $A(x)=\frac{1}{2} x^{2}+x-4$.

$$
\begin{aligned}
& A(x)=\frac{1}{2} x^{2}+x-4 \\
& A(4)=\frac{1}{2}(4)^{2}+(4)-4 \\
& A(4)=\frac{1}{2}(16)+4-4 \\
& A(4)=8+0 \\
& A(4)=8
\end{aligned}
$$

## Tabular

Generate a list of dependent values of $A(x)$ for $A(x)=W(x) \cdot[L(x)+R(x)], W(x)=\frac{1}{2} x-1, L(x)=2 x-1$, and $R(x)=5-x$, when given a list of independent values.

| $x$ | $\begin{gathered} L(x) \\ 2 x-1 \end{gathered}$ | $\begin{aligned} & R(x) \\ & 5-x \end{aligned}$ | $[L(x)+R(x)]$ |  | $\begin{gathered} W(x) \\ \frac{1}{2} x-1 \end{gathered}$ | $W(x) \cdot[L(x)+R(x)]$ | $\begin{gathered} A(x) \\ \frac{1}{2} x^{2}+x-4 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -5 | -11 | 10 | $-11+10$ | -1 | -3.5 | - $-3.5 \bullet-1$ | 3.5 |
| -2 | -5 | 7 | $-5+7$ | 2 | -2 | -2 • 2 | -4 |
| 0 | -1 | 5 | $-1+5$ | 4 | -1 | -1• 4 | -4 |
| 2 |  | 3 | $3+3$ | 6 | 0 | $0 \bullet 6$ | 0 |
| 5 | 9 | 0 | $9+0$ | 9 | 1.5 | $1.5 \bullet 9$ | 13.5 |
| 6 |  | 1 | $11+(-1)$ | 10 | 2 | $2 \cdot 10$ | 20 |

The table shows the dependent value for $A(x)$ evaluated at a particular value of $x$ is equivalent to adding the dependent values for $L(x)$ and $R(x)$ evaluated at the same value of $x$ and multiplying this value by the dependent value for $W(x)$ evaluated at the same value of $x$.

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## Algebraic Reasoning

The table shows the area, $A(x)$, is positive for $x$-values greater than 2 . The side lengths $W(x), L(x)$, and $R(x)$ are positive for $x$-values greater than 2 and less than 5. A reasonable domain for $A(x)$ is all numbers greater than 2 and less than 5 , or $(2,5)$.

## Graphical

Generate dependent values of $A(x)$ for $A(x)=W(x) \cdot[L(x)+R(x)]$, where $W(x)=\frac{1}{2} x-1$, $L(x)=2 x-1$, and $R(x)=5-x$, when given independent values.


The graph shows the dependent value for $A(x)$ evaluated at a particular value of $x$ is equivalent to adding the dependent values for $L(x)$ and $R(x)$ evaluated at the same value of $x$ and multiplying this value by the dependent value for $W(x)$ evaluated at the same value of $x$.

The graph shows the area, $A(x)$, is positive for $x$-values less than -4 and greater than 2 . The side lengths $W(x), S(x)$, and $R(x)$ are all positive for $x$-values greater than 2 and less than 5 . Outside of this interval, the graph shows at least one of the functions has a negative side length; therefore, a reasonable domain for $A(x)$ is all numbers greater than 2 and less than 5 , or $(2,5)$.

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## Algebraic Reasoning

## Marco was shopping in his favorite store

Marco received a $20 \%$ off coupon at his favorite clothing store. He could calculate the cost of anything he bought at the store with the function $f(x)=0.8 x$, where $x$ is the original cost of the item. He realized that he would also have to pay sales tax, which Marco represented with the function $g(x)=0.06(0.8 x)$.

Create a function that Marco can use to calculate $h(x)$, the total cost of an item after discount and tax are applied, given $x$, the original cost of the item.


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## Algebraic Reasoning

## Symbolically:

$f(x)=0.8 x$
$g(x)=0.06(0.8 x)$
$h(x)=f(x)+g(x)$
$h(x)=0.8 x+0.06(0.8 x)$
$h(x)=0.8 x+0.048 x$
$h(x)=0.848 x$

Evaluating $h(x)$ :
$h(x)=f(x)+g(x)$
$h(x)=0.8 x+0.06(0.8 x)$
$h(5)=0.8(5)+0.06(0.8(5)$
$h(5)=4+0.06(4)$
$h(5)=4+0.24$
$h(5)=4.24$

## OR

$h(x)=0.848 x$
$h(5)=0.848(5)$
$h(5)=4.24$
The total cost of a $\$ 5$ item, after a 20\% discount and $6 \%$ sales tax is applied, is \$4.24.

Tabular
Generate a list of dependent values of $h(x)$ for $h(x)=f(x)+g(x), f(x)=0.8 x$, and $g(x)=0.06(0.8 x)$ when given a list of independent values.

| $x$ | $f(x)$ <br> $0.8 x$ | $g(x)$ <br> $0.06(0.8 x)$ | $f(x)+g(x)$ | $h(x)$ <br> $0.848 x$ |
| :---: | :---: | :---: | :---: | :---: |
| 15 | 12 | 0.72 | $12+0.72$ | 12.72 |
| 20 | 16 | 0.96 | $16+0.96$ | 16.96 |
| 30 | 24 | 1.44 | $24+1.44$ | 25.44 |
| 40 | 32 | 1.92 | $32+1.92$ | 33.92 |
| 50 | 40 | 2.40 | $40+2.40$ | 42.40 |
| 100 | 80 | 4.80 | $80+4.80$ | 84.80 |

The table shows the dependent value for $h(x)$ evaluated at a particular value of $x$ is equivalent to adding the dependent values for $f(x)$ and $g(x)$ evaluated at the same value of $x$.

The total cost of a $\$ 50$ item, after a $20 \%$ discount and $6 \%$ sales tax is applied, is $\$ 42.40$.

## Graphical

Generate dependent values of $h(x)$ for $h(x)=f(x)+g(x), f(x)=0.8 x$, and $g(x)=0.06(0.8 x)$ when given independent values.


The graph shows the dependent value for $h(x)$ evaluated at a particular value of $x$ is equivalent to adding the dependent values for $f(x)$ and $g(x)$ evaluated at the same value of $x$.

The total cost of a $\$ 45$ item, after a $20 \%$ discount and $6 \%$ sales tax is applied, is \$38.16.

The symbolic, tabular, and graphical representations show the dependent value for $h(x)$ evaluated at a particular value of $x$ is equivalent to adding the dependent values for $f(x)$ and $g(x)$ evaluated at the same value of $x$.

- Ex:


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## Algebraic Reasoning

The braking distance for a vehicle is the distance in meters required for the vehicle, traveling at a velocity, $v$ in meters per second, to come to a full stop.

The formula for braking distance is composed of a velocity function, $v(x)$, which squares the velocity of the vehicle, $x$, in meters per second at the moment the brakes are applied, and a constant of proportionality function, $k(x)$, which multiplies the friction coefficient of the road and acceleration due to gravity.

Determine the braking distance formula, $d(x)=v(x) \cdot k(x)$, when $v(x)=x^{2}$ and $k(x)=\frac{1}{17.64}$.
The symbolic, tabular, and graphical representations show the dependent value for $d(x)$ evaluated at a particular value of $x$ is equivalent to multiplying the dependent values for $v(x)$ and $k(x)$ evaluated at the same value of $x$.
$\qquad$
$v(x)=x^{2}$
$k(x)=\frac{1}{17.64}$
$d(x)=v(x) \bullet k(x)$
$d(x)=x^{2} \cdot \frac{1}{17.64}$
$d(x)=\frac{x^{2}}{17.64}$

Evaluating $d(x)$ :
$d(x)=v(x) \bullet k(x)$
$d(x)=x^{2} \cdot \frac{1}{17.64}$
$d(30)=(30)^{2} \cdot \frac{1}{17.64}$
$d(30)=900 \cdot \frac{1}{17.64}$
$d(30) \approx 51.02$

## Tabular

Generate a list of dependent values of $d(x)$ for $d(x)=v(x) \bullet k(x), v(x)=x^{2}$, and $k(x)=\frac{1}{17.64}$ when given a list of independent values.

| $x$ | $v(x)$ <br> $x^{2}$ | $k(x)$ <br> $\frac{1}{17.64}$ | $v(x) \bullet k(x)$ | $d(x)$ <br> $\frac{x^{2}}{17.64}$ <br> 0 $0^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{1}{17.64}$ | $0 \bullet \frac{1}{17.64}$ | 0 |  |  |
| 5 | 25 | $\frac{1}{17.64}$ | $25 \bullet \frac{1}{17.64}$ | $\sim 1.4$ |
| 10 | 100 | $\frac{1}{17.64}$ | $100 \bullet \frac{1}{17.64}$ | $\sim 5.7$ |
| 25 | 625 | $\frac{1}{17.64}$ | $625 \bullet \frac{1}{17.64}$ | $\sim 35.4$ |
| 60 | 3,600 | $\frac{1}{17.64}$ | $3,600 \bullet \frac{1}{17.64}$ | $\sim 204.1$ |

The table shows the dependent value for $d(x)$ evaluated at a particular value of $x$ is equivalent to multiplying the dependent values for $v(x)$ and $k(x)$ evaluated at the same value of $x$.

Generate dependent values of $d(x)$ for
$d(x)=v(x) \bullet k(x), v(x)=x^{2}$, and $k(x)=\frac{1}{17.64}$, when given independent values.


The graph shows the dependent value for $d(x)$

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## Algebraic Reasoning

$$
\begin{aligned}
& d(x)=\frac{x^{2}}{17.64} \\
& d(30)=\frac{(30)^{2}}{17.64} \\
& d(30)=\frac{900}{17.64} \\
& d(30) \approx 51.02
\end{aligned}
$$

A distance of approximately 51.02 meters is required for a vehicle traveling at 30 meters per second to come to a full stop after applying the brakes.
The symbolic, tabular, and graphical representations show the dependent value for $d(x)$ evaluated at a particular value of $x$ is equivalent to multiplying the dependent values for $v(x)$ and $k(x)$ evaluated at the same value of $x$.

## Note(s):

- Grade Level(s):
- Algebra I combined like terms and multiplied linear expressions.
- Algebraic Reasoning introduces combining functions using algebraic methods.
- Algebra Il will elaborate on composite functions and their relationship to inverses.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- II.B. Algebraic Reasoning - Manipulating expressions
- II.B.1. Recognize and use algebraic properties, concepts, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).
- II.D. Algebraic Reasoning - Representing relationships
- II.D.1. Interpret multiple representations of equations, inequalities, and relationships.
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions.
- VI.B.2. Algebraically construct and analyze new functions.
- VI.C. Functions - Model real-world situations with functions
- VI.C.1. Apply known functions to model real-world situations.
- VI.C.2. Develop a function to model a situation.

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## Algebraic REASONING

- VII.D. Problem Solving and Reasoning - Real-world problem solving
- VII.D.1. Interpret results of the mathematical problem in terms of the original real-world situation.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.
- VIII.C.2. Create and use representations to organize, record, and communicate mathematical ideas.
- IX.A. Connections - Connections among the strands of mathematics
- IX.A.2. Connect mathematics to the study of other disciplines.
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
- IX.B.2. Understand and use appropriate mathematical models in the natural, physical, and social sciences.

Model a situation using function notation when the output of one function is the input of a second function such as determining a function $h(x)=g(f(x))=1.06(0.8 x)$ for the final purchase price, $h(x)$ of an item with price $x$ dollars representing a $20 \%$ discount, $f(x)=0.8 x$ followed by a $6 \%$ sales tax, $g(x)=1.06 x$.

## Model

A SITUATION USING FUNCTION NOTATION WHEN THE OUTPUT OF ONE FUNCTION IS THE INPUT OF A SECOND FUNCTION SUCH AS DETERMINING A FUNCTION $h(x)=g(f(x))=1.06(0.8 x)$ FOR THE FINAL PURCHASE PRICE, $h(x)$ OF AN ITEM WITH PRICE $x$ DOLLARS REPRESENTING A 20\% DISCOUNT, $f(x)=0.8 x$ FOLLOWED BY A $6 \%$ SALES TAX, $g(x)=1.06 x$

Including, but not limited to:

- Function - a relation in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Composing functions $f(x)$ and $g(x)$
- Notation for composed functions
- $h(x)=g(f(x))$, where $f(x)$ and $g(x)$ are functions
- $h(x)=(f \circ g)(x)$, where $f(x)$ and $g(x)$ are functions
- Dependent values of $f(x)$ are used as the independent values for $g(x)$
- Range of $f(x)$ is the domain of $g(f(x))$
- Ex:


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## Algebraic Reasoning

Trinity drives a small car with a fuel efficiency of 30 miles per gallon. In her city, the average price of gasoline is $\$ 2.75$ per gallon. Trinity wrote the function $f(x)=\frac{x}{30}$ to calculate the amount of gasoline, $f(x)$, in gallons that she would need to drive a distance of $x$ miles. She also wrote the function $g(x)=2.75 x$ to calculate the cost, $g(x)$, of purchasing $x$ gallons of gasoline at $\$ 2.75$ per gallon. Compose a function, $h(x)$, from $f(x)$ and $g(x)$, to determine the cost of the gasoline required to drive $x$ miles. Model the function with a mapping, tabularly, symbolically, and graphically.
(mis)

The mapping models the function $f(x)=\frac{x}{30}$, where the number of miles driven, $x$, is mapped to $f(x)$, the number of gallons of gasoline used.

The mapping models the function $g(x)=2.75 x$, where the number of gallons of gasoline used is mapped to $g(x)$, the cost of the gasoline used. The dependent values from $f(x)$ are used as the independent values of $g(x)$. The range of $f(x)$ is the domain of $g(x)$.


The mapping models the composition of the functions $f(x)=\frac{x}{30}$ and $g(x)=2.75 x$ as $g(f(x))=2.75\left(\frac{x}{30}\right)$, where the number of miles driven, $x$, is mapped to $g(f(x))$, the cost of the gasoline used. The dependent values from $f(x)$ are used as the independent values in $g(f(x))$.

Tabular

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## Mathematics Enhanced TEKS Clarification Document



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| Algebraic Reasoning |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $f(x)=0.8 x$  <br> The graph models the function $f(x)=$ $0.8 x$, where the cost of the item after a $20 \%$ discount has been applied, $f(x)$, is dependent on $x$, the original cost of the item. <br> Note(s): <br> - Grade Level(s): <br> - Algebra I combined like terms and multiplied <br> - Algebraic Reasoning introduces composed through the composition. <br> - Algebra II will extend composition of functio <br> - Precalculus will extend composition of func function of two or more smaller functions. <br> - Various mathematical process standards w <br> - TxCCRS: <br> - II.B. Algebraic Reasoning - Manipulating ex <br> - II.B.1. Recognize and use algebraic prop |  <br> The graph models the function $g(x)=2.75 x$, where the cost of the item after sales tax has been applied, $g(x)$, is dependent on the cost of the item after a $20 \%$ discount has been applied. The dependent values from $f(x)$ are used as the independent values of $g(x)$. The range of $f(x)$ is the domain of $g(x)$. <br> linear and quadratic expressions. functions with an emphasis on following the <br> s and their relationship to inverses. ons to modeling and solving real-world pro <br> I be applied to this student expectation as <br> pressions <br> erties, concepts, and algorithms to combine |  <br> The graph models the composition of the functions $f(x)=0.8 x$ and $g(x)=1.06 x$ as $g(f(x))=1.06(0.8 x)$, where the cost of the item after discount and sales tax has been applied, $g(f(x))$, is dependent on the original cost of the item, $x$. The dependent values from $f(x)$ are used as the independent values in $g(f(x))$. <br> domain values of the original function as th <br> lems and representing a function as a com ppropriate. <br> transform, and evaluate expressions (e.g., | progress <br> site |

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Algebraic Reasoning polynomials, radicals, rational expressions).

- II.D. Algebraic Reasoning - Representing relationships
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- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
- IX.B.3. Know and understand the use of mathematics in a variety of careers and professions.

AR.3F Compare and contrast a function and possible functions that can be used to build it tabularly, graphically, and symbolically such as a quadratic function that results from multiplying two linear functions.

Compare and Contrast

## A FUNCTION AND POSSIBLE FUNCTIONS THAT CAN BE USED TO BUILD IT TABULARLY, GRAPHICALLY, AND SYMBOLICALLY SUCH AS A

 QUADRATIC FUNCTION THAT RESULTS FROM MULTIPLYING TWO LINEAR FUNCTIONSIncluding, but not limited to:

- Function - a relation in which each element of the domain $(x)$ is paired with exactly one element of the range ( $y$ )
- Combined functions
- Added functions
- $h(x)=f(x)+g(x)$, where $f(x)$ and $g(x)$ are functions


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## Algebraic Reasoning

- Dependent values of individual functions are added to determine dependent values of combined function for each particular independent value
- $y$-intercepts of individual functions added to determine $y$-intercept of combined function
- $x$-intercepts of combined function determined by solving $f(x)+g(x)=0$ for $x$
- Ex:

If $h(x)=-\frac{1}{2} x+3$, determine possible functions $f(x)$ and $g(x)$ that can be used to build $h(x)$ using addition. Compare and contrast $f(x), g(x)$, and $f(x)+g(x)$ tabularly, graphically, and symbolically.
Sample response:
The linear function $f(x)=-\frac{1}{2} x$ and the constant function $g(x)=3$ when added result in the linear function $h(x)=-\frac{1}{2} x+3$.


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## Algebraic Reasoning

## Sample response:

The linear functions $f(x)=2 x-1$ and $g(x)=-\frac{5}{2} x+4$ when added result in the linear function $h(x)=-\frac{1}{2} x+3$.

## Symbolic

$f(x)=2 x-1$
$g(x)=-\frac{5}{2} x+4$
$h(x)=f(x)+g(x)$
$h(x)=(2 x-1)+\left(-\frac{5}{2} x+4\right)$
$h(x)=$
$2 x+\left(-\frac{5}{2} x\right)+(-1)+4$
$h(x)=-\frac{1}{2} x+3$
Evaluating $h(x)$ :
$h(x)=-\frac{1}{2} x+3$
$h(4)=-\frac{1}{2}(4)+3$
$h(4)=-2+3$
$h(4)=1$

OR
$f(x)=2 x-1$
$f(4)=2(4)-1$
$f(4)=8-1$
$f(4)=7$

Tabular
Generate a list of dependent values of $h(x)$ for $h(x)=f(x)+g(x), f(x)=2 x-1$, and
$g(x)=-\frac{5}{2} x+4$, when given a list of independent values.

| $x$ | $h(x)$ <br> $-\frac{1}{2} x+3$ | $f(x)+g(x)$ | $f(x)$ <br> $2 x-1$ | $g(x)$ <br> $-\frac{5}{2} x+4$ |
| :---: | :---: | :---: | :---: | :---: |
| -8 | 7 | $-17+24$ | -17 | 24 |
| -1 | $\frac{7}{2}$ | $-3+\frac{13}{2}$ | -3 | $\frac{13}{2}$ |
| 0 | 3 | $-1+4$ | -1 | 4 |
| 3 | $\frac{3}{2}$ | $5+\left(-\frac{7}{2}\right)$ | 5 | $-\frac{7}{2}$ |
| 4 | 1 | $7+(-6)$ | 7 | -6 |
| 6 | 0 | $11+(-11)$ | 11 | -11 |

The table shows the dependent value for $h(x)$ evaluated for a particular value of $x$ is equivalent to adding the dependent values for $f(x)$ and $g(x)$ evaluated for the same value of $x$.

The table shows the $y$-intercept for $h(x), 3$, located at $(0,3)$, is a result of adding the $y$-intercept for $f(x),-1$, located at $(0,-1)$, and the

Graphical
Generate dependent values of $h(x)$ for $h(x)=f(x)+g(x), f(x)=2 x-1$, and $g(x)=-\frac{5}{2} x+4$, when given independent values.


The graph shows the dependent value for $h(x)$ evaluated for a particular value of $x$ is equivalent to adding the dependent values for $f(x)$ and $g(x)$ evaluated for the same value of $x$.

The graph shows the $y$-intercept for $h(x), 3$, located at $(0,3)$, is a result of adding the

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## Algebraic Reasoning

The $x$-intercept for $h(x)$ is 6 , located at the point $(6,0)$.

- Ex:

If $h(x)=x^{2}+2 x-3$, determine possible functions $f(x)$ and $g(x)$ that can be used to build $h(x)$ using addition. Compare and contrast $f(x), g(x)$, and $f(x)+g(x)$ tabularly, graphically, and symbolically.

## Sample response:

The quadratic function $f(x)=x^{2}$ and the linear function $g(x)=2 x-3$ when added result in the quadratic function $h(x)=x^{2}+2 x-3$


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## Algebraic Reasoning

| Algebraic Reasoning |  |  |  |
| :---: | :---: | :---: | :---: |
|  | $h(2)=5$ <br> The symbolic form shows the dependent value for $h(x)$ evaluated for a particular value of $x$ is equivalent to adding the dependent values for $f(x)$ and $g(x)$ evaluated for the same value of $x$. <br> The $x$-intercept(s) for $h(x)$ is determined by solving $f(x)+g(x)=0$ for the variable $x$. $\begin{aligned} & f(x)+g(x)=0 \\ & x^{2}+2 x-3=0 \\ & (x+3)(x-1)=0 \\ & x+3=0 \text { and } x-1=0 \\ & x=-3 \text { and } x=1 \end{aligned}$ <br> The $x$-intercepts for $h(x)$ are -3 , located at the point $(-3,0)$, and 1 , located at the point (1, 0). | $(0,-3)$, is a result of adding the $y$-intercept for $f(x), 0$, located at ( 0,0 ), and the $y$-intercept for $g(x),-3$, located at $(0,-3)$. $\begin{aligned} & h(0)=f(0)+g(0) \\ & h(0)=0+(-3) \\ & h(0)=-3 \end{aligned}$ <br> The table shows an $x$-intercept for $h(x),-3$, located at $(-3,0)$, is a result of adding the $y$-values for $f(x)$ and $g(x)$ evaluated at the same $x$-value, -3 . $\begin{aligned} & h(-3)=f(-3)+g(-3) \\ & h(-3)=9+-9 \\ & h(-3)=0 \end{aligned}$ <br> An $x$-intercept for $h(x)$ is -3 , located at the point $(-3,0)$. <br> The table shows an $x$-intercept for $h(x), 1$, located at $(1,0)$, is a result of adding the $y$-values for $f(x)$ and $g(x)$ evaluated at the same $x$-value, 1 ; $\begin{aligned} & h(1)=f(1)+g(1) \\ & h(1)=1+(-1) \\ & h(1)=0 \end{aligned}$ <br> An $x$-intercept for $h(x)$ is 1 , located at the point $(1,0)$. | The graph shows the $y$-intercept for $h(x),-3$, located at $(0,-3)$, is a result of adding the $y$-intercept for $f(x), 0$, located at $(0,0)$, and the $y$-intercept for $g(x),-3$, located at $(0,-3)$. $\begin{aligned} & h(0)=f(0)+g(0) \\ & h(0)=0+(-3) \\ & h(0)=-3 \end{aligned}$ <br> The graph shows an $x$-intercept for $h(x),-3$, located at $(-3,0)$, is a result of adding the $y$-values for $f(x)$ and $g(x)$ evaluated at the same $x$-value, -1 . $\begin{aligned} & h(-3)=f(-3)+g(-3) \\ & h(-3)=9+-9 \\ & h(-3)=0 \end{aligned}$ <br> An $x$-intercept for $h(x)$ is -3 , located at the point ( $-3,0$ ). <br> The table shows an $x$-intercept for $h(x), 1$, located at $(1,0)$, is a result of adding the $y$-values for $f(x)$ and $g(x)$ evaluated at the same $x$-value, 1. $\begin{aligned} & h(1)=f(1)+g(1) \\ & h(1)=1+(-1) \\ & h(1)=0 \end{aligned}$ <br> An $x$-intercept for $h(x)$ is 1 , located at the point (1, 0). |
|  | Sample response: <br> The quadratic functions $f(x)=3 x^{2}-x+1$ and $g(x)=-2 x^{2}+3 x-4$ when added result in the quadratic function $h(x)=x^{2}+2 x-3$. |  |  |

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## Algebraic Reasoning

The graph shows the $y$-intercept for $h(x),-3$, located at $(0,-3)$, is a result of adding the $y$-intercept for $f(x), 1$, located at $(0,1)$, and the $y$-intercept for $g(x),-4$, located at $(0,-4)$.
$h(0)=f(0)+g(0)$
$h(0)=1+(-4)$
$h(0)=-3$
The graph indicates an $x$-intercept for $h(x),-3$, located at $(-3,0)$, is a result of adding the $y$-values for $f(x)$ and $g(x)$ evaluated at the same $x$-value, -3 .
$h(-3)=f(-3)+g(-3)$
$h(-3)=31+(-31)$
$h(-3)=0$
An $x$-intercept for $h(x)$ is -3 , located at the point $(-3,0)$.
The graph indicates an $x$-intercept for $h(x), 1$, located at $(1,0)$, is a result of adding the $y$-values for $f(x)$ and $g(x)$ evaluated at the same $x$-value, 1 .
$h(1)=f(1)+g(1)$
$h(1)=3+(-3)$
$h(1)=0$
An $x$-intercept for $h(x)$ is 1 , located at the point $(1,0)$.

- Subtracted functions
- $h(x)=f(x)-g(x)$, where $f(x)$ and $g(x)$ are functions
- Dependent values of individual functions are subtracted to determine dependent values of combined function for each particular independent value
- $y$-intercepts of individual functions subtracted to determine $y$-intercept of combined function
- $x$-intercepts of combined function determined by solving $f(x)-g(x)=0$ for $x$
- Ex:

If $h(x)=x^{2}+2 x+3$, determine possible functions $f(x)$ and $g(x)$ that can be used to build $h(x)$ using subtraction. Compare and contrast $f(x), g(x)$, and $f(x)-g(x)$ tabularly, graphically, and symbolically.

Sample response:
The linear function $f(x)=2 x+3$ and the quadratic function $g(x)=-x^{2}$ when subtracted results in the quadratic function
$h(x)=x^{2}+2 x+3$.

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## Algebraic Reasoning

## Symbolic

$f(x)=2 x+3$
$g(x)=-x^{2}$
$h(x)=f(x)-g(x)$
$h(x)=(2 x+3)-\left(-x^{2}\right)$
$h(x)=2 x+3+x^{2}$
$h(x)=x^{2}+2 x+3$
Evaluating $h(x)$ :
$h(x)=x^{2}+2 x+3$
$h(-2)=(-2)^{2}+2(-2)+3$
$h(-2)=4+(-4)+3$
$h(-2)=3$
OR
$f(x)=2 x+3$
$f(-2)=2(-2)+3$
$f(-2)=-4+3$
$f(-2)=-1$
$g(x)=-x^{2}$
$g(-2)=-(-2)^{2}$
$g(-2)=-4$
$h(x)=f(x)-g(x)$
$h(-2)=f(-2)-g(-2)$
$h(-2)=-1-(-4)$
$h(-2)=-1+4$
$h(-2)=3$
The symbolic form shows the dependent value for $h(x)$ evaluated for a particular value of $x$ is equivalent to subtracting the

Tabular
Generate a list of dependent values of $h(x)$ for $h(x)=f(x)-g(x), f(x)=2 x+3$, and $g(x)=-x^{2}$, when given a list of independent values.

| $x$ | $h(x)$ <br> $x^{2}+2 x+3$ | $f(x)-g(x)$ | $f(x)$ <br> $2 x+3$ | $g(x)$ <br> $-x^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| -3 | 6 | $-3-(-9)$ | -3 | -9 |
| -2 | 3 | $-1-(-4)$ | -1 | -4 |
| -1.5 | 2.25 | $0-(-2.25)$ | 0 | -2.25 |
| 0 | 3 | $3-0$ | 3 | 0 |
| 1 | 6 | $5-(-1)$ | 5 | -1 |
| 2 | 11 | $7-(-4)$ | 7 | -4 |

The table shows the dependent value for $h(x)$ evaluated for a particular value of $x$ is equivalent to subtracting the dependent values for $f(x)$ and $g(x)$ evaluated for the same value of $x$.

The table shows the $y$-intercept for $h(x), 3$, located at $(0,3)$, is a result of subtracting the $y$-intercept for $f(x), 3$, located at $(0,3)$, and the $y$-intercept for $g(x)$, 0 , located at $(0,0)$;
$h(0)=f(0)-g(0)$
$h(0)=3+0$
$h(0)=3$
The table shows only positive $y$-values for $h(x)$, indicating $h(x)$ never becomes zero or negative. Since the function $h(x)$ never touches or crosses the $x$-axis, $h(x)$ has no $x$-intercepts.

## Graphical

Generate dependent values of $h(x)$ for $h(x)=f(x)-g(x), f(x)=2 x+3$, and $g(x)=-x^{2}$, when given independent values.


The graph shows the dependent value for $h(x)$ evaluated for a particular value of $x$ is equivalent to subtracting the dependent values for $f(x)$ and $g(x)$ evaluated for the same value of $x$.

The graph shows the $y$-intercept for $h(x), 3$, located at $(0,3)$, is a result of subtracting the $y$-intercept for $f(x), 3$, located at $(0,3)$, and the $y$-intercept for $g(x), 0$, located at ( 0,0 );
$h(0)=f(0)-g(0)$
$h(0)=3+0$
$h(0)=3$
The graph shows only positive $y$-values for $h(x)$ indicating $h(x)$ never becomes zero or negative.

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|  | dependent values for $f(x)$ and $g(x)$ evaluated for the same value of $x$. <br> The $x$-intercept(s) for $h(x)$ is determined by solving $f(x)-g(x)=0$ for $x$. $\begin{aligned} f(x)-g(x) & =0 \\ (2 x+3)-\left(-x^{2}\right) & =0 \\ 2 x+3+x^{2} & =0 \\ x^{2}+2 x+3 & =0 \end{aligned}$ <br> The quadratic $h(x)$ is prime, or not factorable. The discriminant from the quadratic formula will tell how many real solutions $h(x)$ has. $\begin{aligned} & \sqrt{b^{2}-4 a c} \\ & \sqrt{(2)^{2}-4(1)(3)} \\ & \sqrt{4-12} \\ & \sqrt{-8} \end{aligned}$ <br> Since the discriminant is negative, the quadratic function, $h(x)$, has no $x$-intercepts. <br> - Multiplied functions <br> - $h(x)=f(x) \bullet g(x)$, where $f(x)$ <br> - Dependent values of individ value <br> - $y$-intercepts of individual <br> - $x$-intercepts of combined fu <br> - Ex: |  | Since the function $h(x)$ never touches or crosses the $x$-axis, $h(x)$ has no $x$-intercepts. <br> ues of combined function for each particular indepe <br> function |  |
| :---: | :---: | :---: | :---: | :---: |

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## Algebraic Reasoning

If $h(x)=2 x^{2}-5 x-3$, determine possible functions $f(x)$ and $g(x)$ that can be used to build $h(x)$ using multiplication. Compare and contrast $f(x), g(x)$, and $f(x) \bullet g(x)$ tabularly, graphically, and symbolically.

## Sample response:

The linear functions $f(x)=2 x+1$ and $g(x)=x-3$ when multiplied result in the linear function $h(x)=2 x^{2}-5 x-3$.


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Algebraic Reasoning

The symbolic form shows the dependent value for $h(x)$ evaluated for a particular value of $x$ is equivalent to adding the dependent values for $f(x)$ and $g(x)$ evaluated for the same value of $x$.

The $x$-intercept for $h(x)$ is determined by solving $f(x) \cdot g(x)=0$ for $x$.
$f(x) \cdot g(x)=0$
$(2 x+1) \cdot(x-3)=0$
$2 x+1=0$ and $x-3=0$
$2 x=-1$ and $x=3$
$x=-0.5$ and $x=3$ The $x$-intercepts for $h(x)$ are -0.5 , located at the point $(-0.5,0)$, and 3 , located at the point $(3,0)$.

## $h(0)=-3$

The table shows an $x$-intercept for $h(x),-0.5$, located at $(-0.5,0)$, is a result of multiplying the $y$-values for $f(x)$ and $g(x)$ evaluated at the same $x$-value, -0.5 .
$h(-0.5)=f(-0.5) \cdot g(-0.5)$
$h(-0.5)=0 \cdot(-3.5)$
$h(-0.5)=0$
The $x$-intercept for $h(x)$ is -0.5 , located at the point (-0.5, 0).

The table shows an $x$-intercept for $h(x), 3$, located at $(3,0)$, is a result of multiplying the $y$-values for $f(x)$ and $g(x)$ evaluated at the same $x$-value, 3 .
$h(3)=f(3) \bullet g(3)$
$h(3)=7 \bullet 0$
$h(3)=0$
The $x$-intercept for $h(x)$ is 3 , located at the point $(3,0)$.
The table shows the two $x$-intercepts for $h(x)$ result from the product of the linear factors $f(x) \bullet g(x)$, where one of the $y$-values of $f(x)$ or $g(x)$ is 0 . The function $h(x)$ will have two $x$-intercepts, one located at the $x$-intercept of the linear factor $f(x)$, and the other located at the $x$-intercept of the linear factor $g(x)$.

```
y-intercept for g(x), -3, located at (0, -3).
h(0) =f(0) \bulletg(0)
h(0) = 1 • (-3)
h(0) = -3
```

The graph shows an $x$-intercept for $h(x),-0.5$,
located at $(-0.5,0)$, is a result of multiplying the
$y$-values for $f(x)$ and $g(x)$ evaluated at the same
$x$-value, -0.5 .
$h(-0.5)=f(-0.5) \cdot g(-0.5)$
$h(-0.5)=0 \bullet(-3.5)$
$h(-0.5)=0$
The $x$-intercept for $h(x)$ is -0.5 , located at the
point $(-0.5,0)$.

The graph shows an $x$-intercept for $h(x), 3$, located at $(3,0)$, is a result of multiplying the $y$-values for $f(x)$ and $g(x)$ evaluated at the same $x$-value, 3.
$h(3)=f(3) \bullet g(3)$
$h(3)=7 \bullet 0$
$h(3)=0$
The $x$-intercept for $h(x)$ is 3 , located at the point $(3,0)$.

The graph shows the two $x$-intercepts for $h(x)$ result from the product of the linear factors $f(x) \bullet g(x)$, where one of the $y$-values of $f(x)$ or $g(x)$ is 0 . The function $h(x)$ will have two $x$ intercepts, one located at the $x$-intercept of the linear factor $f(x)$, and the other located at the $x$-intercept of the linear factor $g(x)$.

- Divided functions
- $h(x)=f(x) \div g(x)$, or $\frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are functions and $g(x) \neq 0$
- Dependent values of individual functions are divided to determine dependent values of combined function for each particular independent value

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## Algebraic REASONING

- $y$-intercepts of individual functions divided to determine $y$-intercept of combined function
- $x$-intercepts of combined function determined by solving $f(x) \div g(x)=0$, or $\frac{f(x)}{g(x)}=0$, for $x$
- Ex:

If $h(x)=\frac{x^{2}+x-6}{x+3}$, determine possible functions $f(x)$ and $g(x)$ that can be used to build $h(x)$ using division. Compare and contrast $f(x), g(x)$, and $f(x) \div g(x)$ tabularly, graphically, and symbolically.
Sample response:
The quadratic function $f(x)=x^{2}+x+6$ and the linear function $g(x)=x+3$ when divided results in the rational function
$h(x)=\frac{x^{2}+x-6}{x+3}$.
Symbolic Tabular

$$
\begin{aligned}
& f(x)=x^{2}+x+6 \\
& g(x)=x+3 \\
& h(x)=\frac{f(x)}{g(x)} \\
& h(x)=\frac{x^{2}+x-6}{x+3} \quad x \neq-3 \\
& h(x)=\frac{(x+3)(x-2)}{x+3}, x \neq-3
\end{aligned}
$$

Evaluating $h(x)$ :

$$
\begin{aligned}
& h(x)=\frac{(x+3)(x-2)}{x+3} \\
& h(1)=\frac{((1)+3)((1)-2)}{(1)+3} \\
& h(1)=\frac{(4)(-1)}{4} \\
& h(1)=-1
\end{aligned}
$$

Generate a list of dependent values of $h(x)$ for

$$
h(x)=f(x) \div g(x), f(x)=x^{2}+x+6, \text { and }
$$ $g(x)=x+3$, when given a list of independent variables

Graphical

Generate dependent values of $h(x)$ for $h(x)=f(x) \div g(x), f(x)=x^{2}+x+6$, and $g(x)=x+3$, when given independent values


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The graph shows the output or $y$-value for $h(x)$ evaluated for a particular value of $x$ is equivalent to multiplying the outputs or $y$-values for $f(x)$ and $g(x)$ evaluated for the same value of

The graph shows the $y$-intercept for $h(x),-2$, located at $(0,-2)$, is a result of multiplying the $y$-intercept for $f(x),-6$, located at $(0,-6)$, and the
$y$-intercept for $g(x), 3$, located at $(0,3)$;
$h(0)=\frac{f(0)}{g(0)}$
$h(0)=\frac{-6}{3}$
$h(0)=-2$
The graph shows the $x$-intercept for $h(x), 2$, located at $(2,0)$, is a result of dividing the $y$-values for $f(x)$ and $g(x)$ evaluated at the same $x$-value, 2 ;
$h(2)=f(2) \div g(2)$
$h(2)=0 \div 5$
$h(2)=0$
The $x$-intercept for $h(x)$ is 2 , located at the point $(2,0)$.

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| Algebraic Reasoning |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & -4 t-8=0 \text { and } 4 t-16=0 \\ & -4 t=8 \text { and } 4 t=16 \\ & t=-2 \text { and } t=4 \end{aligned}$ <br> Since the first $x$-intercept, $x=-2$, occurs 2 seconds before the object is launched, the solution is extraneous and therefore discarded. <br> The other $x$-intercept, $x=4$, represents the time the object hits the ground. | $t$ $f(t)$ <br> $-4 t-8$ $g(t)$ <br> $4 t-16$ $f(t) \bullet g(t)$ $h(t)$ <br> $-16 x^{2}+32 x+128$ <br> -3 4 -28 $4 \bullet(-28)$ -112 <br> -2 0 -24 $0 \bullet(-24)$ 0 <br> -1 -4 -20 $-4 \bullet(-20)$ 80 <br> 0 -8 -16 $-8 \bullet(-16)$ 128 <br> 1 -12 -12 $-12 \bullet(-12)$ 144 <br> 2 -16 -8 $-16 \bullet(-8)$ 128 <br> 3 -20 -4 $-20 \bullet(-4)$ 80 <br> 4 -24 0 $-24 \bullet 0$ 0 <br> 5 -28 4 $-28 \bullet 4$ -112 <br> The table shows the $x$-intercept for $f(t)$ is -2 , located at the point $(-2,0)$, and the $x$-intercept for $g(t)$ is 4 , located at the point $(4,0)$. Since at least one of the $y$-values at the $x$-intercepts for $f(t)$ and $g(t)$ is 0 , the product, $f(t) \bullet g(t)$, representing the $y$-value of the $x$-intercept for $h(t)$ will also be 0 . The $x$-intercepts of $f(t)$ and $g(t)$ are the $x$-intercepts of $h(t)$; therefore $h(t)$ has two $x$-intercepts, -2 , located at the point $(-2,0)$, and 4 , located at the point $(4,0)$. <br> The $x$-intercept of $t=-2$ is discarded as extraneous because a negative time value indicates the object hits the ground 2 seconds before the object is launched. This solution is not reasonable within the context of the problem situation. <br> The $x$-intercept of $t=4$ is reasonable within the context of the problem situation. The ball hits the ground 4 seconds after the object is launched with a velocity of 32 feet per second from a height of 128 feet above the ground. |  |  |  |  |  |
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## Algebraic Reasoning

Graphical
Determine the $x$-intercept(s) of $h(t)$ and its linear factors, $f(t)$ and $g(t)$, to find the time the object hits the ground.


The table shows the $x$-intercept for $f(t)$ is -2 , located at the point $(-2,0)$, and the $x$-intercept for $g(t)$ is 4 , located at the point $(4,0)$. Since at least one of the $y$-values at the $x$-intercepts for $f(t)$ and $g(t)$ is 0 in the product, $f(t) \bullet g(t)$, representing the $y$-value of the $x$-intercept for $h(t)$ will also be 0 . The $x$-intercepts of the linear factors $f(t)$ and $g(t)$ are the $x$-intercepts of $h(t)$; therefore $h(t)$ has two $x$-intercepts, -2 , located at the point $(-2,0)$, and 4 , located at the point $(4,0)$.

The $x$-intercept of $t=-2$ is discarded as extraneous because a negative time value indicates the object hits the ground 2 seconds before the object is launched. This solution is not reasonable within the context of the problem situation.

The $x$-intercept of $t=4$ is reasonable within the context of the problem situation. The ball hits the ground 4 seconds after the object is launched with a velocity of 32 feet per second from a height of 128 feet above the ground.

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Algebraic Reasoning
Note(s):

- Grade Level(s):
- Grade 8 introduced linear functions
- Algebra I introduced quadratic and exponential functions.
- Algebra I connected linear factors with the roots of quadratic functions.
- Algebraic Reasoning introduces the absolute value, cubic, rational, square root, cube root, and logarithmic functions.
- Algebraic Reasoning reinforces connections among linear, quadratic, and cubic functions through multiple representations of key attributes of the functions and their component parts.
- Algebra Il will introduce and expand on higher degree polynomial functions.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- II.B. Algebraic Reasoning - Manipulating expressions
- II.B.1. Recognize and use algebraic properties, concepts, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).
- II.D. Algebraic Reasoning - Representing relationships
- II.D.1. Interpret multiple representations of equations, inequalities, and relationships.
- VI.A. Functions - Recognition and representation of functions
- VI.A.2. Recognize and distinguish between different types of functions.
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions.
- VI.B.2. Algebraically construct and analyze new functions
- VI.C. Functions - Model real-world situations with functions
- VI.C.2. Develop a function to model a situation.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VII.D. Problem Solving and Reasoning - Real-world problem solving
- VII.D.1. Interpret results of the mathematical problem in terms of the original real-world situation.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
- VIII.A.2. Use mathematical language to represent and communicate the mathematical concepts in a problem.
- VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations,
- VIII.B.2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.2. Create and use representations to organize, record, and communicate mathematical ideas.
- VIII.C.3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

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## Algebraic Reasoning

- IX.A. Connections - Connections among the strands of mathematics
- IX.A.2. Connect mathematics to the study of other disciplines.
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.

AR. 4

AR.4A a variety of ways, including real-world situations. The student is expected to

Connect tabular representations to symbolic representations when adding, subtracting, and multiplying polynomial functions arising from mathematical and real-world situations such as applications involving surface area and volume.

Connect
TABULAR REPRESENTATIONS TO SYMBOLIC REPRESENTATIONS WHEN ADDING, SUBTRACTING, AND MULTIPLYING POLYNOMIAL FUNCTIONS ARISING FROM MATHEMATICAL AND REAL-WORLD SITUATIONS SUCH AS APPLICATIONS INVOLVING SURFACE AREA AND VOLUME

Including, but not limited to:

- Function - a relation in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Polynomial function - a relation that can be represented by a monomial or sum of monomials, not including variables in the denominator or under a radical, in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Adding polynomial functions
- Tabular representations
- Evaluation of the independent values or $x$-values of individual functions are added to determine the dependent values or $y$-values of the combined function.
- Symbolic representations
- Evaluation of the independent values or $x$-values of the combined function determines the dependent values or $y$-values of the combined function.
- $h(x)=f(x)+g(x)$, where $f(x)$ and $g(x)$ are functions
- Connection of tabular representations to symbolic representations
- Sum of the dependent values or $y$-values of individual functions is equivalent to the evaluation of the independent values or $x$-values of the combined function.
- Mathematical situations
- Ex:

Given the functions $f(x)=3 x^{2}+6 x-30$ and $g(x)=-2 x^{3}-4 x+5$,
a) Use tabular representations to determine the dependent values of $f(x)=3 x^{2}+6 x-30$ and the dependent values of $g(x)=-2 x^{3}-4 x+5$, and use these values to find the values of $h(x)$, where $h(x)=f(x)+g(x)$.

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## Algebraic Reasoning

b) Use symbolic representations to determine $h(x)$, where $h(x)=f(x)+g(x)$, and use $h(x)$ to determine the dependent values of the combined functions.
c) Connect the results of the tabular representations to the symbolic representations.
a) Tabular

Sample response:
$f(x)=3 x^{2}+6 x-30$
$g(x)=-2 x^{3}-4 x+5$
$h(x)=f(x)+g(x)$

| $x$ | $f(x)$ | $g(x)$ | $h(x)$ <br> $f(x)+g(x)$ | $h(x)$ |
| :---: | :---: | :---: | :---: | :---: |
| -2 | -30 | 29 | $-30+29$ | -1 |
| -1 | -33 | 11 | $-33+11$ | -22 |
| 0 | -30 | 5 | $-30+5$ | -25 |
| 1 | -21 | -1 | $-21+(-1)$ | -22 |
| 2 | -6 | -19 | $-6+(-19)$ | -25 |
| 3 | 15 | -61 | $15+(-61)$ | -46 |

b) Symbolic

Sample response:

$$
\begin{aligned}
& f(x)=3 x^{2}+6 x-30 \\
& g(x)=-2 x^{3}-4 x+5
\end{aligned}
$$

$$
h(x)=f(x)+g(x)
$$

$$
h(x)=\left(3 x^{2}+6 x-30\right)+\left(-2 x^{3}-4 x+5\right)
$$

$$
h(x)=3 x^{2}+6 x-30-2 x^{3}-4 x+5
$$

$$
h(x)=-2 x^{3}+3 x^{2}+2 x-25
$$

| $x$ | $-2 x^{3}+3 x^{2}+2 x-25$ | $h(x)$ |
| :---: | :---: | :---: |
| -2 | $-2(-2)^{3}+3(-2)^{2}+2(-2)-25$ | -1 |
| -1 | $-2(-1)^{3}+3(-1)^{2}+2(-1)-25$ | -22 |
| 0 | $-2(0)^{3}+3(0)^{2}+2(0)-25$ | -25 |
| 1 | $-2(1)^{3}+3(1)^{2}+2(1)-25$ | -22 |
| 2 | $-2(2)^{3}+3(2)^{2}+2(2)-25$ | -25 |
| 3 | $-2(3)^{3}+3(3)^{2}+2(3)-25$ | -46 |

c) Connections

Sample response:
Using a tabular representation, the evaluation of the independent values or $x$-values of $f(x)$ and $g(x)$ are added to determine the dependent values or $y$-values of the combined function, $h(x)=f(x)+g(x)$.

Using a symbolic representation, the evaluation of the independent values or $x$-values of the combined function, $h(x)=-2 x^{3}+3 x^{2}+2 x-25$, determines the dependent values or $y$-values of the combined function, $h(x)$.

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## Algebraic REASONING

Using both representations, the sum of the dependent values or $y$-values of $h(x)=f(x)+g(x)$ is equivalent to the evaluation of the independent values or $x$-values of the combined function, $h(x)=-2 x^{3}+3 x^{2}+2 x-25$.

- Real-world situations
- Ex:

A square pyramid is given where the length of one side of the base is $4 x$ and the slant height is $2 x-1$.


1. Write polynomial functions for $B(x)$, the area of the base of the square pyramid, and $L(x)$, the lateral surface area of the square pyramid.
a) Use tabular representations to determine the dependent values of $B(x)$ and the dependent values of $L(x)$, and use these values to find $T(x)$, the total surface area of the square pyramid, where $T(x)=B(x)+L(x)$.
b) Use symbolic representations to determine $T(x)$, where $T(x)=B(x)+L(x)$, and then use $T(x)$ to determine the dependent values of the combined functions.
c) Connect the results of the tabular representations to the symbolic representations.

## Sample response

1. Area of the base of the square pyramid
$B(x)=$ (length of side of base)(length of side of base)
$B(x)=(4 x)(4 x)$
$B(x)=16 x^{2}$

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## Algebraic Reasoning

The local ice cream parlor sells 3 sizes of ice cream cones: small, medium, and large. The height of the cone is 5 inches tall for all three sizes of cones. The owner determines that the ice cream cone consists of a hemisphere (half of a sphere) of ice cream (the scoop) and the filled cone. He uses a chart to list the volume of ice cream he sells for each size of ice cream cone:

| Size | Total Ice cream |
| :---: | :---: |
| Small <br> 2 inch <br> radius | 37.70 cubic <br> inches |
| Medium <br> 2.5 inch <br> radius | 65.46 cubic <br> inches |
| Large <br> 3 inch <br> radius | 103.67 cubic <br> inches |

A math student wants to show that one formula could be used to evaluate the total volume of each ice cream cone. Using the listed volumes of total ice cream, the formula for the volume of a cone $\left(C(r)=\frac{1}{3} \pi r^{2} h\right)$, and the formula for the volume of a sphere $\left(S(r)=\frac{4}{3} \pi r^{3}\right)$, show that using a single formula will predict the volumes of the ice cream cones on the chart.

1. Write polynomial functions for $S(r)$, the volume of a hemi-sphere, and $C(r)$, the volume of a cone.
a) Use tabular representations to determine the dependent values of $S(r)$ and the dependent values of $C(r)$, and use these values to find $T(r)$, the total volume of the hemi-sphere and cone, where $T(r)=S(r)+C(r)$.
b) Use symbolic representations to determine $T(r)$, where $T(r)=S(r)+C(r)$, and then use $T(r)$ to determine the dependent values of the combined functions.
c) Connect the results of the tabular representations to the symbolic representations.

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## ALgebraic ReAsoning

c) Connections:

Sample response:
Using a tabular representation, the evaluation of the independent values or $r$-values of $S(r)$ and $C(r)$ are added to determine the dependent values of the combined function, $T(r)=S(r)+C(r)$.

Using a symbolic representation, the evaluation of the independent values of the combined function,
$T(r)=\frac{2}{3} \pi r^{3}+\frac{5}{3} \pi r^{2}$, determines the dependent values of the combined function, $T(r)$.
The total volume of the ice cream in an ice cream cone can be represented by the sum of the dependent values of $T(r)=S(r)+C(r)$, which is equivalent to the evaluation of the independent values or $r$-values of the combined function, $T(r)=\frac{2}{3} \pi r^{3}+\frac{5}{3} \pi r^{2}$.

- Subtracting polynomial functions
- Tabular representations
- Evaluation of the independent values of individual functions are subtracted to determine the dependent values of the combined function.
- Symbolic representations
- Evaluation of the independent values of the combined function determines the dependent values of the combined function.
- $h(x)=f(x)-g(x)$, where $f(x)$ and $g(x)$ are functions
- Connection of tabular representations to symbolic representations
- Difference of the dependent values of individual functions is equivalent to the evaluation of the independent values of the combined function.
- Mathematical situations
- Ex:

Given the functions $f(x)=3 x^{2}+6 x-30$ and $g(x)=-2 x^{3}-4 x+5$,
a) Use tabular representations to determine the dependent values of $f(x)=3 x^{2}+6 x-30$ and the dependent values of $g(x)=-2 x^{3}-4 x+5$, and use these values to find the values of $h(x)$, where $h(x)=f(x)-g(x)$.
b) Use symbolic representations to determine $h(x)$, where $h(x)=f(x)-g(x)$, and use $h(x)$ to determine the dependent values of the combined functions.
c) Connect the results of the tabular representations to the symbolic representations.

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## Algebraic Reasoning

a) Tabular

Sample response:
$f(x)=3 x^{2}+6 x-30$
$g(x)=-2 x^{3}-4 x+5$
$h(x)=f(x)-g(x)$
b) Symbolic

Sample response:
$f(x)=3 x^{2}+6 x-30$
$g(x)=-2 x^{3}-4 x+5$
$h(x)=f(x)-g(x)$
$h(x)=\left(3 x^{2}+6 x-30\right)-\left(-2 x^{3}-4 x+5\right)$
$h(x)=3 x^{2}+6 x-30+2 x^{3}+4 x-5$
$h(x)=2 x^{3}+3 x^{2}+10 x-35$

| $x$ | $f(x)$ | $g(x)$ | $f(x)-g(x)$ | $h(x)$ |
| :---: | :---: | :---: | :---: | :---: |
| -2 | -30 | 29 | $-30-29$ | -59 |
| -1 | -33 | 11 | $-33-11$ | -44 |
| 0 | -30 | 5 | $-30-5$ | -35 |
| 1 | -21 | -1 | $-21-(-1)$ | -20 |
| 2 | -6 | -19 | $-6-(-19)$ | 13 |
| 3 | 15 | -61 | $15-(-61)$ | 76 |


|  | $h(x)$ |  |
| :---: | :---: | :---: |
| $x$ | $2 x^{3}+3 x^{2}+10 x-35$ | $h(x)$ |
| -2 | $2(-2)^{3}+3(-2)^{2}+10(-2)-35$ | -59 |
| -1 | $2(-1)^{3}+3(-1)^{2}+10(-1)-35$ | -44 |
| 0 | $2(0)^{3}+3(0)^{2}+10(0)-35$ | -35 |
| 1 | $2(1)^{3}+3(1)^{2}+10(1)-35$ | -20 |
| 2 | $2(2)^{3}+3(2)^{2}+10(2)-35$ | 13 |
| 3 | $2(3)^{3}+3(3)^{2}+10(3)-35$ | 76 |

c) Connections

Sample response
Using a tabular representation, the evaluation of the independent values of $f(x)$ and $g(x)$ are subtracted to determine the dependent values of the combined function, $h(x)=f(x)-g(x)$.

Using a symbolic representation, the evaluation of the independent values of the combined function, $h(x)=2 x^{3}+3 x^{2}+10 x-35$, determines the dependent values of the combined function, $h(x)$.

Using both representations, the difference of the dependent values of $h(x)=f(x)-g(x)$ is equivalent to the evaluation of the independent values of the combined function, $h(x)=2 x^{3}+3 x^{2}+10 x-35$.

- Real-world situations
- Ex:

Daryl's toy is in the shape of rectangular prism and has a cylindrical hole drilled through the center of the prism along the height of the prism. The base of each prism is a square with a side length that is 3 times the diameter of the cylindrical hole. The height of the prism is 5 times the diameter of the hole.

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## Algebraic Reasoning



Determine a function Daryl could write to find the amount of plastic to use in the mold for each toy, if the diameter of the hole on each base is $x$ units.

1. Write polynomial functions for $R(x)$, the volume of the rectangular prism, and $C(x)$, the volume of the cylindrical hole.
a) Use tabular representations to determine the dependent values of $R(x)$ and the dependent values of $C(x)$, and use these values to find $V(x)$, the volume of the rectangular prism not including the cylindrical hole, where $V(x)=R(x)-C(x)$.
b) Use symbolic representations to determine $V(x)$, where $V(x)=R(x)-C(x)$, and then use $V(x)$ to determine the dependent values of the combined functions.
c) Connect the results of the tabular representations to the symbolic representations.

## Sample response:

1. Volume of the rectangular prism:
$R(x)=B h$, where $B$ represents the area of the square base and $h$ represents the height of the prism
$R(x)=(3 x)(3 x)(5 x)$
$R(x)=\left(9 x^{2}\right)(5 x)$
$R(x)=45 x^{3}$
The volume of the rectangular prism is $B(x)=45 x^{3}$.
Volume of the cylindrical hole:
$C(x)=B h$, where $B$ represents the area of the circular base and $h$ represents the height of the cylinder

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- Multiplication of polynomial functions
- Tabular representations
- Evaluation of the independent values of individual functions are multiplied to determine the dependent values of the combined function.
- Symbolic representations
- Evaluation of the independent values of the combined function determines the dependent values of the combined function.
- $h(x)=f(x) \bullet g(x)$, where $f(x)$ and $g(x)$ are functions
- Connection of tabular representations to symbolic representations
- Product of the dependent values of individual functions is equivalent to the evaluation of the independent values of the combined function.
- Mathematical situations
- Ex:

Given the functions $f(x)=3 x^{2}+6 x-30$ and $g(x)=4 x+5$,
a) Use tabular representations to determine the dependent values of $f(x)=3 x^{2}+6 x-30$ and the dependent values of $g(x)=4 x+5$, and use these values to find the value of $h(x)$, where $h(x)=f(x) \bullet g(x)$.
b) Use symbolic representations to determine $h(x)$, where $h(x)=f(x) \bullet g(x)$, and use $h(x)$ to determine the dependent values of the combined functions.
c) Connect the results of the tabular representations to the symbolic representations.

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## Algebraic Reasoning

a) Tabular

Sample response:
$f(x)=3 x^{2}+6 x-30$
$g(x)=4 x+5$
$h(x)=f(x) \bullet g(x)$

## b) Symbolic

Sample response:
$f(x)=3 x^{2}+6 x-30$
$g(x)=4 x+5$
$h(x)=f(x) \bullet g(x)$
$h(x)=\left(3 x^{2}+6 x-30\right)(4 x+5)$
$h(x)=3 x^{2}(4 x+5)+6 x(4 x+5)-30(4 x+5)$
$h(x)=12 x^{3}+15 x^{2}+24 x^{2}+30 x-120 x-150$
$h(x)=12 x^{3}+39 x^{2}-90 x-150$

| $x$ | $f(x)$ | $g(x)$ | $h(x)$ <br> $f(x) \bullet g(x)$ | $h(x)$ |
| :---: | :---: | :---: | :---: | :---: |
| -2 | -30 | -3 | $-30 \bullet(-3)$ | 90 |
| -1 | -33 | 1 | $-33 \bullet 1$ | -33 |
| 0 | -30 | 5 | $-30 \bullet 5$ | -150 |
| 1 | -21 | 9 | $-21 \bullet 9$ | -189 |
| 2 | -6 | 13 | $-6 \bullet 13$ | -78 |
| 3 | 15 | 17 | $15 \bullet 17$ | 255 |


| $x$ | $h(x)$ |  |
| :---: | :---: | :---: |
| $x$ | $12 x^{3}+39 x^{2}-90 x-150$ | $h(x)$ |
| -2 | $12(-2)^{3}+39(-2)^{2}-90(-2)-150$ | 90 |
| -1 | $12(-1)^{3}+39(-1)^{2}-90(-1)-150$ | -33 |
| 0 | $12(0)^{3}+39(0)^{2}-90(0)-150$ | -150 |
| 1 | $12(1)^{3}+39(1)^{2}-90(1)-150$ | -189 |
| 2 | $12(2)^{3}+39(2)^{2}-90(2)-150$ | -78 |
| 3 | $12(3)^{3}+39(3)^{2}-90(3)-150$ | 255 |

c) Connections

Sample response:
Using a tabular representation, the evaluation of the independent values of $f(x)$ and $g(x)$ are multiplied to determine the dependent values of the combined function, $h(x)=f(x) \bullet g(x)$.

Using a symbolic representation, the evaluation of the independent values of the combined function, $h(x)=12 x^{3}+39 x^{2}-90 x-150$, determines the dependent values of the combined function, $h(x)$.

Using both representations, the product of the dependent values of $h(x)=f(x) \bullet g(x)$ is equivalent to the evaluation of the independent values of the combined function,
$h(x)=12 x^{3}+39 x^{2}-90 x-150$.

- Real-world situations
- Ex:


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## Algebraic Reasoning

All Things Clay makes custom-sized clay bricks for decorative uses. The dimensions of each brick are determined in terms of the width of the brick. When ordering, the customer states the width of each brick being purchased, in inches. The length of each brick will be 3 times the width and the height will be 4 inches longer than the width. The company would like to write a function to determine the volume of clay required for each brick based on the given width of the brick.

1. Write polynomial functions for $W(x)$, the width of the brick, $L(x)$, the length of the brick in terms of the width, and $H(x)$, the height of the brick in terms of the width.
a) Use tabular representations to determine the dependent values of $W(x), L(x)$, and $H(x)$, and use these values to find $V(x)$, the volume of the brick, where $V(x)=W(x) \bullet L(x) \bullet H(x)$.
b) Use symbolic representations to determine $V(x)$, where $V(x)=W(x) \bullet L(x) \bullet H(x)$, and then use $V(x)$ to determine the dependent values of the combined functions.
c) Connect the results of the tabular representations to the symbolic representations.

## Sample response:

1. The independent variable $x$ represents the requested width in inches.


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## Algebraic Reasoning

| 3 | 3 | 9 | 7 | $3 \cdot 9 \cdot 7$ | 189 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 4 | 12 | 8 | $4 \cdot 12 \cdot 8$ | 384 |
| 5 | 5 | 15 | 9 | $5 \cdot 15 \cdot 9$ | 675 |
| 6 | 6 | 18 | 10 | $6 \cdot 18 \cdot 10$ | 1,080 |


| 3 | $3(3)^{3}+12(3)^{2}$ | 189 |
| :---: | :---: | :---: |
| 4 | $3(4)^{3}+12(4)^{2}$ | 384 |
| 5 | $3(5)^{3}+12(5)^{2}$ | 675 |
| 6 | $3(6)^{3}+12(6)^{2}$ | 1,080 |

c) Connect

Sample response:
Using a tabular representation, the evaluation of the independent values of $W(x), L(x)$, and $H(x)$ are multiplied to determine the dependent values of the combined function, $V(x)=W(x) \bullet L(x) \bullet H(x)$.

Using a symbolic representation, the evaluation of the independent values of the combined function, $V(x)=3 x^{3}+12 x^{2}$, determines the dependent values of the combined function, $V(x)$.

The volume of the clay brick can be represented by the product of the dependent values of
$V(x)=W(x) \bullet L(x) \bullet H(x)$, which is equivalent to the evaluation of the independent values of the combined function, $V(x)=3 x^{3}+12 x^{2}$.

Note(s):

- Grade Level(s):
- Algebra I evaluated polynomial expressions up to degree two.
- Algebra I added, subtracted, and multiplied polynomial expressions up to degree two.
- Algebraic Reasoning introduces operating on polynomial functions up to degree three.
- Algebra Il will continue to evaluate and operate on polynomial expressions, equations, and functions up to an $n^{\text {th }}$ degree.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- II.A. Algebraic Reasoning - Identifying expressions and equations
- II.A.1. Explain the difference between expressions and equations.
- II.B. Algebraic Reasoning - Manipulating expressions
- II.B.1. Recognize and use algebraic properties, concepts, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions.
- VI.B.2. Algebraically construct and analyze new functions.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving


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## Algebraic REASONING

- VII.A.1. Analyze given information.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.2. Use mathematical language to represent and communicate the mathematical concepts in a problem.
- VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.
- VIII.C.2. Create and use representations to organize, record, and communicate mathematical ideas.
- IX.A. Connections - Connections among the strands of mathematics
- IX.A.2. Connect mathematics to the study of other disciplines.
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
- IX.B.3. Know and understand the use of mathematics in a variety of careers and professions.


## AR.4B

Compare and contrast the results when adding two linear functions and multiplying two linear functions that are represented tabularly, graphically, and symbolically.

Compare, Contrast

## THE RESULTS WHEN ADDING TWO LINEAR FUNCTIONS AND MULTIPLYING TWO LINEAR FUNCTIONS THAT ARE REPRESENTED

 TABULARLY, GRAPHICALLY, AND SYMBOLICALLYIncluding, but not limited to:

- Function - a relation in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Linear function - a relationship with a constant rate of change represented by a graph that forms a straight line in which each element of the input $(x)$ is paired with exactly one element of the output $(y)$
- Results of adding two linear functions
- New linear function, $h(x)$, formed, where $h(x)=f(x)+g(x)$
- If $f(x)=a x+b$, and $g(x)=c x+d$, then $h(x)=(a+c) x+(b+d)$.
- Sum of dependent values of $f(x)$ and $g(x)$ is equal to the dependent value of $h(x)$ for any given independent value, $x$
- Sum of $y$-intercepts of $f(x)$ and $g(x)$ is equal to the $y$-intercept of $h(x)$
- Sum of $x$-intercepts of $f(x)$ and $g(x)$ has no relationship to the $x$-intercept of $h(x)$
- $x$-intercept for $h(x)$ occurs at $x$-value where $f(x)$ and $g(x)$ have opposite dependent values
- When $f(x)=0$ (the $x$-intercept for $f(x)$ ), then $g(x)$ and $h(x)$ intersect and have the same dependent values.
- When $g(x)=0$ (the $x$-intercept for $g(x)$ ), then $f(x)$ and $h(x)$ intersect and have the same dependent values.
- Sum of the slopes of $f(x)$ and $g(x)$ is the slope of $h(x)$.
- Ex:


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## Algebraic Reasoning

Given $f(x)=2 x+4$ and $g(x)=x-3$,
a) Use a tabular representation to determine the dependent values of $f(x)=2 x+4$ and the dependent values of $g(x)=x-3$, and use these values to find the values of $h(x)$, where $h(x)=f(x)+g(x)$
b) Use a graphical representation to determine the dependent values of $f(x)=2 x+4$ and the dependent values of $g(x)=x-3$, and use these values to find the values of $h(x)$, where $h(x)=f(x)+g(x)$.
c) Use a symbolic representation to determine $h(x)$, where $h(x)=f(x)+g(x)$, and use $h(x)$ to determine the dependent values of the combined functions.
d) Compare and contrast the results of adding linear functions using tabular, graphical, and symbolic representations.

| a) Tabular <br> Sample response: $\begin{aligned} & f(x)=2 x+4 \\ & g(x)=x-3 \\ & h(x)=f(x)+g(x) \end{aligned}$ |  |  |  |  | b) Graphical <br> Sample response: $\begin{aligned} & f(x)=2 x+4 \\ & g(x)=x-3 \\ & h(x)=f(x)+g(x) \end{aligned}$  $h(x)=f(x)+g(x)=3 x+1$ | $\begin{aligned} & \text { c) Syn } \\ & f(x)=2 \\ & g(x)=x \\ & h(x)=f \\ & h(x)=( \\ & h(x)=2 \\ & h(x)=2 \\ & h(x)=3 \end{aligned}$ | olic $\begin{aligned} & +4 \\ & -3 \\ & +9(x) \\ & x+4)+(x \\ & +4+x-3 \\ & +x+4-3 \\ & +1 \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x$ | $f(x)$ | $g(x)$ | $\begin{gathered} h(x) \\ f(x)+g(x) \end{gathered}$ | $h(x)$ |  | $x$ | $h(x)$ $3 x+1$ | $h(x)$ |
| 0 | 4 | -3 | $4+(-3)$ | 1 |  | 0 | $3(0)+1$ | 1 |
| 2 | 8 | -1 | $8+(-1)$ | 7 |  | 2 | $3(2)+1$ | 7 |
| 5 | 14 | 2 | $14+2$ | 16 |  | 5 | $3(5)+1$ | 16 |
| 9 | 22 | 6 | $22+6$ | 28 |  | 9 | $3(9)+1$ | 28 |
| 13 | 30 | 10 | $30+10$ | 40 |  | 13 | $3(13)+1$ | 40 |
| 20 | 44 | 17 | $44+17$ | 61 |  | 20 | $3(20)+1$ | 61 |

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## Algebraic Reasoning

## d) Compare and contrast

Sample response:
Using a tabular representation, the evaluation of the independent values of $f(x)$ and $g(x)$ are added to determine the dependent values of the combined function, $h(x)=f(x)+g(x)$.

Using a symbolic representation, the evaluation of the independent values of the combined function, $h(x)=3 x+1$, determines the dependent values of the combined function, $h(x)$.

Using both representations, the sum of the dependent values of $h(x)=f(x)+g(x)$ is equivalent to the evaluation of the independent values of the combined function, $h(x)=3 x+1$.

The addition of two linear functions, $f(x)$ and $g(x)$, results in a new linear function, $h(x)$, where $h(x)=f(x)+g(x)$. The slope of $h(x)$ results from the addition of the slopes of $f(x)$ and $g(x)$. The $y$-intercept of $h(x)$ results from the addition of the $y$-intercepts of $f(x)$ and $g(x)$.

The functions $f(x)$ and $g(x)$ have opposite values when evaluated using the $x$-intercept of $h(x)$.
The graphs of $g(x)$ and $h(x)$ intersect when $f(x)=0$ (the $x$-intercept for $f(x)$ ), which is equivalent to the dependent values of $g(x)$ and $h(x)$ being the same when evaluated using the $x$-intercept of $f(x)$. Similarly, the graphs of $f(x)$ and $h(x)$ intersect when $g(x)=0$ (the $x$-intercept for $g(x)$ ), which is equivalent to the dependent values of $f(x)$ and $h(x)$ being the same when evaluated using the $x$-intercept of $g(x)$.

- Results of multiplying two linear functions
- New quadratic function, $h(x)$, formed, where $h(x)=f(x) \bullet g(x)$
- If $f(x)=a x+b$ and $g(x)=c x+d$, where $a \neq 0$ and $c \neq 0$, then $h(x)=(a c) x^{2}+(a d+b c) x+b d$.
- $f(x)$ and $g(x)$ are factors in the product $h(x)$
- Product of dependent values of $f(x)$ and $g(x)$ is equal to the dependent value of $h(x)$ for any given independent value, $x$
- Product of $y$-intercepts of $f(x)$ and $g(x)$ is equal to the $y$-intercept of $h(x)$
- $x$-intercepts of product function, $h(x)$, are $x$-intercepts of factor functions $f(x)$ and $g(x)$
- Midpoint between $x$-intercepts of $h(x)$ is $x$-coordinate of the vertex
- Midpoint between $x$-intercepts of $h(x)$ is $x$-coordinate used in the vertical equation of the axis of symmetry
- Ex:

Given $f(x)=2 x+4$ and $g(x)=x-3$
a) Use a tabular representation to determine the dependent values of $f(x)=2 x+4$ and the dependent values of $g(x)=x-3$, and use these values to find the value of $h(x)$, where $h(x)=f(x) \bullet g(x)$.

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## Algebraic Reasoning

b) Use a graphical representation to determine the dependent values of $f(x)=2 x+4$ and the dependent values of $g(x)=x-3$, and use these values to find the value of $h(x)$, where $h(x)=f(x) \bullet g(x)$.
c) Use a symbolic representation to determine $h(x)$, where $h(x)=f(x) \bullet g(x)$, and use $h(x)$ to determine the dependent values of the combined functions.
d) Compare and contrast the results of adding linear functions using tabular, graphical, and symbolic representations.
a) Tabular

Sample response:
$f(x)=2 x+4$
$g(x)=x-3$
$h(x)=f(x) \bullet g(x)$

| $x$ | $f(x)$ | $g(x)$ | $h(x)$ <br> $f(x) \bullet g(x)$ | $h(x)$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 4 | -3 | $4 \bullet(-3)$ | -12 |
| 2 | 8 | -1 | $8 \bullet(-1)$ | -8 |
| 5 | 14 | 2 | $14 \bullet 2$ | 28 |
| 9 | 22 | 6 | $22 \bullet 6$ | 132 |
| 13 | 30 | 10 | $30 \bullet 10$ | 300 |
| 20 | 44 | 17 | $44 \bullet 17$ | 748 |

b) Graphical

Sample response:
$f(x)=2 x+1$
$g(x)=x-3$
$h(x)=f(x) \bullet g(x)$
$g(x)$

$$
h(x)=f(x)+g(x
$$

c) Symbolic
$f(x)=2 x+4$
$g(x)=x-3$
$h(x)=f(x) \cdot g(x)$
$h(x)=(2 x+4)(x-3)$
$h(x)=(2 x)(x-3)+(4)(x-3)$
$h(x)=2 x^{2}-6 x+4 x-12$
$h(x)=2 x^{2}-2 x-12$

| $x$ | $h(x)$ |  |
| :---: | :---: | :---: |
| $x$ | $2 x^{2}-2 x-12$ | $h(x)$ |
| 0 | $2(0)^{2}-2(0)-12$ | -12 |
| 2 | $2(2)^{2}-2(2)-12$ | -8 |
| 5 | $2(5)^{2}-2(5)-12$ | 28 |
| 9 | $2(9)^{2}-2(9)-12$ | 132 |
| 13 | $2(13)^{2}-2(13)-12$ | 300 |
| 20 | $2(20)^{2}-2(20)-12$ | 748 |

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## Algebraic ReAsoning

## d) Compare and contrast

Sample response:
Using a tabular representation, the evaluation of the independent values of $f(x)$ and $g(x)$ are multiplied to determine the dependent values of the combined function, $h(x)=f(x) \bullet g(x)$.

Using a symbolic representation, the evaluation of the independent values of the combined function, $h(x)=2 x^{2}-2 x-12$, determines the dependent values of the combined function, $h(x)$.

Using both representations, the product of the dependent values of $h(x)=f(x) \bullet g(x)$ is equivalent to the evaluation of the independent values of the combined function, $h(x)=2 x^{2}-2 x-12$.

The product of two linear functions, $f(x)$, and $g(x)$, results in a quadratic function, $h(x)$, where $h(x)=f(x) \bullet g(x)$. The $y$-intercept of $h(x)$ results from the multiplication of the $y$-intercepts of $f(x)$ and $g(x)$

The $x$-intercepts of $f(x)$ and $g(x)$ are the $x$-intercepts of $h(x)$. The $x$-coordinate of the vertex of the graph of $h(x)$ is the midpoint of the $x$-intercepts of $f(x)$ and $g(x)$, or the $x$-intercepts of $h(x)$. The axis of symmetry is located at the midpoint of the $x$-intercepts of $f(x)$ and $g(x)$, or the $x$-intercepts of $h(x)$.

Note(s):

- Grade Level(s):
- Algebra I formalized polynomial addition and multiplication up to second degree polynomials.
- Algebra I related the roots to the linear factors of a quadratic function.
- Algebraic Reasoning extends the relationships between linear addends and factors and their resulting functions using multiple representations.
- Algebra Il will introduce the relationships between linear and quadratic factors of polynomial expressions and equations.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- II.A. Algebraic Reasoning - Identifying expressions and equations
- II.A.1. Explain the difference between expressions and equations.
- II.B. Algebraic Reasoning - Manipulating expressions
- II.B.1. Recognize and use algebraic properties, concepts, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).
- VI.A. Functions - Recognition and representation of functions
- VI.A.2. Recognize and distinguish between different types of functions.
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions.
- VI.B.2. Algebraically construct and analyze new functions.


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## Algebraic Reasoning

- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.2. Use mathematical language to represent and communicate the mathematical concepts in a problem.
- VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
- VIII.B.2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.
- VIII.C.2. Create and use representations to organize, record, and communicate mathematical ideas.
- VIII.C.3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Determine the quotient of a polynomial function of degree three and of degree four when divided by a polynomial function of degree one and of degree two when represented tabularly and symbolically.

## Determine

## THE QUOTIENT OF A POLYNOMIAL FUNCTION OF DEGREE THREE AND OF DEGREE FOUR WHEN DIVIDED BY A POLYNOMIAL FUNCTION OF DEGREE ONE AND OF DEGREE TWO WHEN REPRESENTED TABULARLY AND SYMBOLICALLY

Including, but not limited to:

- Function - a relation in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Polynomial function - a relation that can be represented by a monomial or sum of monomials, not including variables in the denominator or under a radical, in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Various methods for dividing polynomial functions with remainders of 0
- Factoring
- Factor polynomials in numerator and denominator
- Box Method
- Factor Table
- Grouping
- Simplification of common factors in numerator and denominator that divide to 1
- Degree three polynomial functions
- Degree three polynomial function divided by degree one polynomial function - Ex:


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## Algebraic REASONING

Determine the quotient, $h(x)$, by dividing the degree three polynomial function, $f(x)=3 x^{3}-15 x^{2}$, by the degree one polynomial function, $g(x)=x-5$, using factoring and division to 1 .


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## Algebraic REASONING

## 6. Simplify the equation. The right fraction in the product is equal

to 1 , so the remaining quotient is, $h(x)=3 x^{2}$.

$$
\begin{aligned}
& h(x)=\frac{3 x^{2}}{1} \bullet 1 \\
& h(x)=3 x^{2}
\end{aligned}
$$

7. Name the quotient and undefined value(s) for the variable, when they exist.

Therefore, $h(x)=\left(3 x^{3}-15 x^{2}\right) \div(x-5)$ results in $h(x)=3 x^{2}, x \neq 5$.

- Degree three polynomial function divided by degree two polynomial function
- Ex:

| Determine the quotient, $h(x)$, by dividing the degree three polynomial function, $f(x)=x^{3}-5 x^{2}+x-5$, by the degree two polynomial function, $g(x)=6 x^{2}+6$, using factoring and division to 1 . |  |
| :---: | :---: |
| Sample response: |  |
| 1. Write the quotient in fraction form, with the dividend, $f(x)$, in the numerator and the divisor, $g(x)$, in the denominator. | $\begin{aligned} & f(x)=x^{3}-5 x^{2}+x-5 \\ & g(x)=6 x^{2}+6 \\ & h(x)=f(x) \div g(x) \\ & h(x)=\left(x^{3}-5 x^{2}+x-5\right) \div\left(6 x^{2}+6\right) \\ & h(x)=\frac{x^{3}-5 x^{2}+x-5}{6 x^{2}+6} \end{aligned}$ |
| 2. Check for a GCF other than 1 in the numerator and denominator and factor out any GCF other than 1. In this case, factor out the GCF in the denominator, 6. The GCF of the numerator is 1 . | $h(x)=\frac{x^{3}-5 x^{2}+x-5}{6\left(x^{2}+1\right)}$ |
| 3. Check if each factor in the numerator and denominator can be further factored using an appropriate factor method. <br> a. In this case, terms are grouped in the numerator using parentheses and a GCF of $x^{2}$ is factored out of the first grouped terms. <br> b. Factor out the common term, $(x-5)$. | $\begin{aligned} & h(x)=\frac{\left(x^{3}-5 x^{2}\right)+(x-5)}{6\left(x^{2}+1\right)} \\ & h(x)=\frac{x^{2}(x-5)+(x-5)}{6\left(x^{2}+1\right)} \\ & h(x)=\frac{\left(x^{2}+1\right)(x-5)}{6\left(x^{2}+1\right)} \end{aligned}$ |
| 4. Prior to simplifying, determine if any value(s) for a variable would result in division by zero. If that occurs, then the value(s) would be excluded. | Since there are no values of $x$ that will make the denominator $6\left(x^{2}+1\right)$ equal to 0 , then there are no domain restrictions for $h(x)$. |
| 5. Using factors that appear in both the numerator and denominator, rewrite as a product of two fractions; the right | $h(x)=\frac{(x-5)}{6} \cdot \frac{\left(x^{2}+1\right)}{\left(x^{2}+1\right)}$ |

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## Algebraic REASONINg

fraction includes the factors that appeared in both the
numerator and denominator, $\frac{\left(x^{2}+1\right)}{\left(x^{2}+1\right)}$, and the left fraction
includes the remaining factors, $\frac{(x-5)}{6}$.
6. The right fraction in the product is equal to 1 , so the quotient is
the remaining fraction, $h(x)=\frac{(x-5)}{6}$.

$$
h(x)=\frac{(x-5)}{6}
$$

7. Name the quotient and undefined value(s) for the variable, when they exist.

Therefore, $h(x)=\left(x^{3}-5 x^{2}+x-5\right) \div\left(6 x^{2}+6\right)$ results in $h(x)=\frac{x-5}{6}$.

- Degree four polynomial functions
- Degree four polynomial function divided by degree one polynomial function
- Ex:

Determine the quotient, $h(x)$, by dividing the degree four polynomial function, $f(x)=x^{4}-16$, by the degree one polynomial function, $g(x)=x-2$, using factoring and division to 1 . Sample response:

1. Write the quotient in fraction form, with the dividend, $f(x)$, in the numerator and the divisor, $g(x)$, in the denominator.


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## Algebraic Reasoning

3. Check if each factor in the numerator and denominator can be factored using an appropriate factor
a. In this case, the perfect square trinomial in the numerator, $x^{4}-16$, is factored to $\left(x^{2}+4\right)\left(x^{2}-4\right)$.
b. In this case, the perfect square trinomial in the numerator, $x^{2}-4$, is factored to $(x+2)(x-2)$.
4. Prior to simplifying, determine if any value(s) for a variable would result in division by zero. If that occurs, then the value(s) would be excluded.

$$
\begin{aligned}
& h(x)=\frac{\left(x^{2}+4\right)\left(x^{2}-4\right)}{(x-2)} \\
& h(x)=\frac{\left(x^{2}+4\right)(x+2)(x-2)}{(x-2)}
\end{aligned}
$$

Since $x=2$ results in division by 0 ,

5. Using factors that appear in both the numerator and denominator, rewrite as a product of two fractions; the left fraction includes remaining factors after removal of like factors, $\frac{\left(x^{2}+4\right)(x+2)}{1}$ appeared in both the numerator and denominator, $\frac{(x-2)}{(x-2)}$.
6. The right fraction in the product is equal to 1 , so the quotient is


$$
\begin{aligned}
& h(x)=\frac{\left(x^{2}+4\right)(x+2)}{1} \cdot 1 \\
& h(x)=\left(x^{2}+4\right)(x+2) \\
& h(x)=x^{3}+2 x^{2}+4 x+8
\end{aligned}
$$

7. Name the quotient and undefined value(s) for the variable, when they exist.

The function representing the quotient is $h(x)=x^{3}+2 x^{2}+4 x+8, x \neq 2$.

- Degree four polynomial function divided by degree two polynomial function
- Ex:

Determine the quotient, $h(x)$, by dividing the degree four polynomial function, $f(x)=2 x^{4}+5 x^{2}-12$, by the degree two polynomial function, $g(x)=2 x^{2}+8$, using factoring and division to 1 .
Sample response:

1. Write the quotient in fraction form, with the dividend, $f(x)$, in the numerator and the divisor, $g(x)$, in the denominator.

$$
\begin{aligned}
& f(x)=2 x^{4}+5 x^{2}-12 \\
& g(x)=2 x^{2}+8 \\
& h(x)=f(x) \div g(x)
\end{aligned}
$$

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## Algebraic Reasoning



- Long division
- Missing terms in series represented by adding a zero term
- Identification of independent values or $x$-values, in the divisor function which make the quotient function undefined
- Connections between long division of whole numbers and long division of polynomial functions


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## Algebraic ReAsoning

- Degree three polynomial functions
- Degree three polynomial function divided by degree one polynomial function
- Ex:

| Determine the quotient, $h(x)$, by dividing the degree three polynomial function, $f(x)=5 x^{3}-20 x^{2}$, by the degree one polynomial function, $g(x)=x-4$, using long division. |  |
| :---: | :---: |
| Sample response: |  |
| 1. Set up the division problem as whole number long division. Write terms in descending order of degree. | $\begin{aligned} & f(x)=5 x^{3}-20 x^{2} \\ & g(x)=x-4 \\ & h(x)=f(x) \div g(x) \\ & h(x)=\left(5 x^{3}-20 x^{2}\right) \div(x-4) \\ & x - 4 \longdiv { 5 x ^ { 3 } - 2 0 x ^ { 2 } } \end{aligned}$ |
| 2. Divide the leading term, $5 x^{3}$, in the dividend by the leading term, $x$, in the divisor, getting $5 x^{2}$ in the quotient. Place $5 x^{2}$ above the $x^{2}$ term of the dividend. | $x - 4 \longdiv { 5 x ^ { 3 } - 2 0 x ^ { 2 } }$ |
| 3. Multiply $5 x^{2}$ through the divisor and place the products under the dividend, lining up like terms. | $\begin{array}{r} 5 x^{2} \\ x - 4 \longdiv { 5 x ^ { 3 } - 2 0 x ^ { 2 } } \\ 5 x^{3}-20 x^{2} \\ \hline \end{array}$ |
| 4. Subtract the terms by adding their opposites (changing the signs) which will cause the leading terms to add to 0 . | $\begin{array}{r} 5 x^{2} \\ x - 4 \longdiv { 5 x ^ { 3 } - 2 0 x ^ { 2 } } \\ \frac{-\left(5 x^{3}-20 x^{2}\right)}{0} \end{array}$ |
| 5. Since the divisor is the denominator of the fraction, factor the denominator, if possible. Determine if any value(s) for a variable would result in division by zero. If that occurs, then the value(s) would be excluded. | Since $x-4$ results in division by 0 , $h(x)=\frac{5 x^{3}-20 x^{2}}{x-4}$ is not defined when $x=4$. |
| 6. Name the quotient and undefined value(s) for the variable, when they exist. | The function representing the quotient is $h(x)=5 x^{2}, x \neq 4$. |

- Degree three polynomial function divided by degree two polynomial function
- Ex:


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## Algebraic Reasoning

Determine the quotient, $h(x)$, by dividing the degree three polynomial function, $f(x)=7 x^{3}+2 x^{2}-89 x+60$, by the degree two polynomial function, $g(x)=7 x^{2}-26 x+15$, using long division.
Sample response:

| 1. Set up the division problem as whole number long division. Write terms in descending order of degree. | $\begin{aligned} & f(x)=7 x^{3}+2 x^{2}-89 x+60 \\ & g(x)=7 x^{2}-26 x+15 \\ & h(x)=f(x) \div g(x) \\ & h(x)=\left(7 x^{3}+2 x^{2}-89 x+60\right) \div\left(7 x^{2}-26 x+15\right) \\ & \left.7 x^{2}-26 x+15\right) \longdiv { 7 x ^ { 3 } + 2 x ^ { 2 } - 8 9 x + 6 0 } \end{aligned}$ |
| :---: | :---: |
| 2. Divide the leading term, $7 x^{3}$, in the dividend by the leading term, $7 x^{2}$, in the divisor, getting $x$ in the quotient. Place $x$ above the $x$ term of the dividend. | $7 x ^ { 2 } - 2 6 x + 1 5 \longdiv { 7 x ^ { 3 } + 2 x ^ { 2 } - 8 9 x + 6 0 }$ |
| 3. Multiply $x$ through the divisor and place the products under the dividend, lining up like terms. | $\begin{gathered} 7 x ^ { 2 } - 2 6 x + 1 5 \longdiv { 7 x ^ { 3 } + 2 x ^ { 2 } - 8 9 x + 6 0 } \\ \underline{7 x^{3}-26 x^{2}+15 x} \end{gathered}$ |
| 4. Subtract the terms by adding their opposites (changing the signs) which will cause the leading terms to add to 0 . Bring down the next term. | $\begin{array}{r} 7 x ^ { 2 } - 2 6 x + 1 5 \longdiv { 7 x ^ { 3 } + 2 x ^ { 2 } - 8 9 x + 6 0 } \\ \frac{-\left(7 x^{3}-26 x^{2}+15 x\right)}{28 x^{2}-104 x+60} \end{array}$ |
| 5. Divide the leading term, $28 x^{2}$, in the dividend by the leading term, $7 x^{2}$, in the divisor, getting 4 in the quotient. Place 4 above the 4 term of the dividend. | $\begin{array}{rl} 7 x^{2}-26 x+15 & x+4 \\ 7 x^{3}+2 x^{2}-89 x+60 \\ \frac{-\left(7 x^{3}-26 x^{2}+15 x\right)}{28 x^{2}-104 x}+60 \end{array}$ |

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## Algebraic Reasoning



- Degree four polynomial functions
- Degree four polynomial function divided by degree one polynomial function
- Ex:

Determine the quotient, $h(x)$, by dividing the degree four polynomial function, $f(x)=x^{4}-2 x^{2}+1$, by the degree one polynomial function, $g(x)=x-1$, using long division.

Sample response:

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## Algebraic Reasoning

7. Name the quotient and undefined value(s) for the variable, $\quad$ The function representing the quotient is when they exist.

$$
h(x)=x^{3}+x^{2}-x-1, x \neq 1 .
$$

- Degree four polynomial function divided by degree two polynomia
- Ex:

Determine the quotient, $h(x)$, by dividing the degree four polynomial function, $f(x)=x^{4}+2 x^{3}-7 x^{2}-20 x-12$, by the degree two polynomial function, $g(x)=x^{2}-x-6$, using long division.

Sample response:

1. Set up the division problem as with whole number long division. Write terms in descending order of degree.

$$
\begin{aligned}
& f(x)=x^{4}+2 x^{3}-7 x^{2}-20 x-12 \\
& g(x)=x^{2}-x-6 \\
& h(x)=f(x) \div g(x) \\
& h(x)=\left(x^{4}+2 x^{3}-7 x^{2}-20 x-12\right) \div\left(x^{2}-x-6\right)
\end{aligned}
$$

$$
x ^ { 2 } - x - 6 \longdiv { x ^ { 4 } + 2 x ^ { 3 } - 7 x ^ { 2 } - 2 0 x - 1 2 }
$$

2. Divide the leading term, $x^{4}$, in the dividend by the leading term, $x^{2}$, in the divisor, getting $x^{2}$ in the quotient. Place $x^{2}$ above the $x^{2}$ term of the dividend.
3. Multiply $x^{2}$ through the divisor and place the products under the dividend, lining up like terms.
4. Subtract the terms by adding their opposites (changing the signs) which will cause the leading term to add to 0 . Bring down the next term(s)

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## Algebraic Reasoning


6. Since the divisor is the denominator of the fraction, factor the denominator, if possible. Determine if any value(s) for a variable would result in division by zero. If that occurs, then the value(s) would be excluded.

Since $x=-2$ and $x=3$ result in division by 0 ,
$h(x)=\frac{x^{4}+2 x^{3}-7 x^{2}-20 x-12}{x^{2}-x-6}$, which is
equivalent to $h(x)=\frac{x^{4}+2 x^{3}-7 x^{2}-20 x-12}{(x-3)(x+2)}$,
is not defined when $x=3$ and $x=-2$.
7. Name the quotient and undefined value(s) for the variable, when they exist.

The function representing the quotient is
$h(x)=x^{2}+2 x^{3}-7 x-20 x-12, x \neq 3$ and $x \neq-2$.

- Synthetic division
- Algorithm for dividing a polynomial function by a known zero of the function working only with the coefficients of the dividend function and a zero of the dividend function
- Synthetic division only allows division by a linear, or first-degree, divisor.
- Zero of a polynomial function corresponds to one of the linear factors of the function
- Linear factor represented by $x-c$
- Zero of a polynomial function determined by solving $x-c=0$.
- Degree three polynomial function divided by degree one polynomial in the form $x-c$
- Ex:

Determine the quotient, $h(x)$, by dividing the degree third polynomial function, $f(x)=5 x^{3}-20 x^{2}$, by the degree one
polynomial function, $g(x)=x-4$, using synthetic division.

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## Algebraic Reasoning



- Degree four polynomial function by degree one polynomial in the form $x-c$
- Ex:

Determine the quotient, $h(x)$, by dividing the degree four polynomial function, $f(x)=x^{4}-5 x^{2}+3 x+10$, by the degree one polynomial function, $g(x)=x+2$, using synthetic division.

1. Check that the zero of the divisor function, $g(x)$, is also a zero of the dividend function, $f(x)$. In this case, $g(-2)=0$ and $f(-2)=0$; therefore $x=-2$ is a zero of $f(x)$. Let $c$ represent the value of this zero.

$f(x)=x^{4}-5 x^{2}+3 x+10$
$g(x)=x+2$
$h(x)=f(x) \div g(x)$
Divisor function:
$g(x)=x+2$
$g(-2)=(-2)+2$
$g(-2)=0$
$f(x)=x^{4}-5 x^{2}+3 x+10$
$f(-2)=(-2)^{4}-5(-2)^{2}+3(-2)+10$
$f(-2)=16-5(4)-6+10$
$f(-2)=16-20+4$
$f(-2)=0$
$x=-2$ represents a zero of $f(x)$.

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## ALgebraic Reasoning

| 2. In a box in the upper left corner, record the known zero of $f(x), c$. Next to the box, write the coefficients of the dividend in descending order of degree. For terms missing in the descending order, fill in zeros as place holders for their coefficients. Leave a blank space beneath the coefficients for later use and draw a line. | $\begin{array}{llllll} -2 & 1 & 0 & -5 & 3 & 10 \\ & & & & & \\ \hline \end{array}$ |
| :---: | :---: |
| 3. In the first column, bring down the first number and place it below the line. |  |
| 4. Multiply the number brought down in the first column by the divisor, -2 , and place the product in the blank space under the second number. Add the two numbers in the second column and put the result under the line. | $\begin{array}{cccccc} -2 & 1 & 0 & -5 & 3 & 10 \\ & -2 & & & \\ \hline & 1 & -2 & & & \end{array}$ |
| 5. Multiply the divisor, -2 , by the result in the second column and place the product in the blank space under the third number. Add the two numbers in the third column and put the result under the line. | -2 1 0 -5 3 10 <br>   -2 4   <br>  1 -2 -1   |
| 6. Repeat this process until the last column has a result. Box in the last sum. | $-2]$ 1 0 -5 3 10 <br>   -2 4 2 -10 <br>  1 -2 -1 5 $\underline{0}$ |
| 7. The bottom line represents the quotient of the division. The first digit is the first coefficient of the first term with the power decreased by one degree from the original dividend. The next numbers, except the last one, are the coefficients of the next terms with descending degree. The last number is the remainder. | $h(x)=x^{3}-2 x^{2}-x+5$ with remainder of 0. |
| 8. Since the divisor is the denominator of the fraction, factor the denominator, if possible. Determine if any value(s) for a variable would result in division by zero. If that occurs, then the value(s) would be excluded. | Since $x=-2$ results in division by 0 , $h(x)=\frac{x^{4}-5 x^{2}+3 x+10}{x+2}$ is not defined when $x=-2$. |
| 9. Name the quotient and undefined value(s) for the variable, when they exist. | The quotient function is $h(x)=x^{3}-2 x^{2}-x+5, x \neq-2$. |

- Determine the quotient when dividing degree three and degree four polynomial functions by degree one and degree two polynomial functions
- Tabular representation
- Determination of dependent values of the quotient function by dividing dependent values of the dividend and divisor functions


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## Mathematics Enhanced TEKS Clarification Document

## ALgEBRAIC REASONING

- Undefined values in quotient function when divisor function has a value of zero
- Patterns in finite differences
- Degree one polynomial function
- Pattern in $y$-values have a common first difference
- First differences - a list of common differences between the first successive dependent values, $\Delta y$, when first differences between successive independent values, $\Delta x$, are also a common difference
- Linear functions have a dependent first common difference when there is an independent first common difference.
- Linear function representations Linear function representations
- Standard form, $a x+b y=c$
- Slope-intercept form, $y=m x+b$, where $m$ represents the slope and $b$ represents the $y$-intercept
- Slope of a linear function is represented by $m=\frac{\text { change in } y \text {-values }}{\text { change in } x \text {-values }}=\frac{\Delta y}{\Delta x}=\frac{\text { first common difference }}{\Delta x}$.
- $(x, y)$ represents a point on the line
- $b$ represents the $y$-intercept, $(0, b)$
- Point-slope form, $y-y_{1}=m\left(x-x_{1}\right)$, where $m$ is the slope and $\left(x_{1}, y_{1}\right)$ represents a point of the line
- Slope of a linear function is represented by $m=\frac{\text { change in } y \text {-values }}{\text { change in } x \text {-values }}=\frac{\Delta y}{\Delta x}=\frac{\text { first common difference }}{\Delta x}$.
- $\left(x_{1}, y_{1}\right)$ represents a point on the line
- Degree two polynomial function
- Pattern in $y$-values have a non-zero common second difference
- Second differences - a list of common differences between the second successive dependent values, $\Delta y$, when the differences between first successive independent values, $\Delta x$, are also constant.
- Quadratic functions have a dependent second common difference when there is an independent first common difference.
- Quadratic function representation
- Standard form, $f(x)=a x^{2}+b x+c$, where $a, b$, and $c$ are rational numbers
- First difference between the $y$-values when $x=0$ and $x=1$ is $a+b$.
- Second common differences between the $y$-values are related to a by second common difference is $2 a$ when $\Delta x=1$.
- Value of $c$ represents $(0, c)$ or the $y$-intercept
- Degree three polynomial function
- Pattern in $y$-values have a non-zero common third difference
- Third differences - a list of common differences between the third successive $y$-values, $\Delta y$, when first differences between successive $x$ values, $\Delta x$, are also a common difference
- Cubic functions have a dependent third common difference when there is an independent first common difference.
- Third differences that result in a constant difference or a common difference represent a cubic function.
- Cubic function representation
- Standard form, $f(x)=a x^{3}+b x^{2}+c x+d$, where $a, b, c$, and $d$ are rational numbers


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## Algebraic Reasoning

- First difference between the $y$-values when $x=0$ and $x=1$ is $a+b+c$.
- Second difference between the first two differences of $y$-values between $x=0$ and $x=1$, and $x=1$ and $x=2$, is $6 a+2 b$
- Third common differences between the $y$-values are related to a by third common difference $=6 a$, when $\Delta x=1$.
- Value of $d$ represents $(0, d)$ or the $y$-intercept
- Ex:

Given the polynomial functions $f(x)=x^{3}-4 x^{2}-25 x+100$ and $g(x)=x^{2}+x-20$,
a) Use a tabular representation to determine the dependent values of $f(x)=x^{3}-4 x^{2}-25 x+100$ and the dependent values of $g(x)=x^{2}+x-20$, and use these values to find the dependent values of $h(x)$, where $h(x)=f(x) \div g(x)$.
b) Use this tabular representation to determine the quotient, $h(x)$, where $h(x)=f(x) \div g(x)$
a) Represent

$$
\begin{aligned}
& f(x)=x^{3}-4 x^{2}-25 x+100 \\
& g(x)=x^{2}+x-20 \\
& h(x)=f(x) \div g(x)
\end{aligned}
$$

| $x$ | $f(x)$ | $g(x)$ | $h(x)$ <br> $f(x) \div g(x)$ | $h(x)$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 100 | -20 | $100 \div(-20)$ | -5 |
| 1 | 72 | -18 | $72 \div(-18)$ | -4 |
| 2 | 42 | -14 | $42 \div(-14)$ | -3 |
| 3 | 16 | -8 | $16 \div(-8)$ | -2 |
| 4 | 0 | 0 | $0 \div 0$ | undefined |
| 5 | 0 | 10 | $0 \div 10$ | 0 |
| 6 | 22 | 22 | $22 \div 22$ | 1 |

b) Determine the polynomial function, $h(x)$, that represents the quotient, $f(x) \div g(x)$, of the polynomial functions $f(x)$ and $g(x)$.

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## Algebraic Reasoning

Since the differences between successive $x$-values, $\Delta x$, is
1, use patterns in the finite differences between
successive $y$-values, $\Delta y$, to determine $h(x)$.


Since a constant finite difference or common difference results from the first differences between successive $y$-values, $\Delta y$, the pattern demonstrates a linear function relationship.

The quotient, $h(x)$, is not defined when $x=4$; therefore $h(x)$ is a linear function that is not defined when $x=4$.

The general form of a linear function is $h(x)=m x+b$. Use patterns in the finite differences to determine the values of $m$ and $b$.

Slope: $m=1$
$y$-intercept: $b=-5$
$y=m x+b$
$y=(1) x+(-5)$
$y=x-5$
The quotient function is $h(x)=x-5$.
The tabular representation may not show the $x$-values for which a quotient function will have undefined $y$-values. The table of values for $h(x)$ shows that $h(4)$ is undefined, but did not show that $h(-5)$ is also undefined. The tabular representation may be limited in identifying undefined $y$-values when $x$ is a rational or irrational number.
-
Given the functions $f(x)=-12 x^{3}-119 x^{2}-155 x+700$ and $g(x)=-3 x-20$, determine the polynomial function, $h(x)$, that represents the quotient, $f(x) \div g(x)$, of the dependent values $f(x)$ and $g(x)$.

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## Algebraic Reasoning

Since the differences between successive $x$-values, $\Delta x$, is
1, use patterns in the finite differences between successive $y$-values, $\Delta y$, to determine $h(x)$.

| $\boldsymbol{x}$ | $\boldsymbol{h}(\boldsymbol{x})$ |
| :---: | :---: |
| -1 | -44 |
| 0 | -35 |
| 1 | -18 |
| 2 | 7 |
| 3 | 40 |
| 4 | 81 |$>$

$\Delta y$

## Dependent

 First DifferenceDependent Second Difference


Since a constant finite difference or common difference results from the second differences between successive $y$-values, $\Delta y$, the pattern demonstrates a quadratic function relationship.

The general form of a quadratic function is $h(x)=a x^{2}+b x+c$. Use patterns in the finite differences to determine the values of $a, b$, and $c$.

The second common differences between the $y$-values are related to $a$ by second common difference is $2 a$, when $\Delta x=1$.
$8=2 a$

The first difference between the $y$-values for $x=0$ and $x=1$
is $a+b$. This value in the table is 17 .
$a+b=17$
(4) $+b=17$
$b=13$
The value of the $y$-intercept is $c$. This value in the table is -35 . $c=-35$
$h(x)=a x^{2}+b x+c$
$h(x)=(4) x^{2}+(13) x+(-35)$
$h(x)=4 x^{2}+13 x-35$
The quotient function is $h(x)=4 x^{2}+13 x-35$
The tabular representation may not show the $x$-values for which a quotient function will have undefined $y$-values. The table of values for $h(x)$ does not show that $h\left(-6 \frac{2}{3}\right)$ is an undefined $y$-value. The tabular representation may be limited in identifying undefined $y$-values when $x$ is a rational or irrational number.

- Regression method using technology
- Ex:


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## Algebraic Reasoning

Given a table of values for the functions $f(x)=x^{3}-4 x^{2}-25 x+100$ and $h(x)=x-5$, determine the function, $q(x)$, that represents the quotient of the dependent values using a regression method.

| $x$ | $x^{3}-4 x^{2}-25 x+100$ | $h(x)$ <br> $x-5$ | $f(x) \div h(x)$ |
| :---: | :---: | :---: | :---: |
| 0 | 100 | -5 | $100 \div(-5)=-20$ |
| 1 | 72 | -4 | $72 \div(-4)=-18$ |
| 2 | 42 | -3 | $42 \div(-3)=-14$ |
| 3 | 16 | -2 | $16 \div(-2)=-8$ |
| 4 | 0 | -1 | $0 \div(-1)=0$ |

Use technology to determine the quotient function by generating a regression model.


Sample technology response:
Quadratic Regression
$\operatorname{regEQ}(x)=x^{2}+x-20$
$a=1$
$b=1$
$c=-20$
$q(x)=x^{2}+x-20$

The quotient of $f(x) \div h(x)$ is $q(x)=x^{2}+x-20$

- Symbolic representation
- Evaluation of the independent values of the quotient function determines the dependent values of the divided function
- $h(x)=f(x) \div g(x)$, where $f(x)$ and $g(x)$ are functions


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## Algebraic Reasoning

- Quotient function, $h(x)=f(x) \div g(x)$, will have undefined dependent values when the dependent value of the divisor function is zero, or $g(x)=0$.
- Connection of tabular representations to symbolic representations
- Quotient of the dependent values of individual functions is equivalent to the evaluation of the independent values of the quotient function.
- Ex:

Determine the quotient, $h(x)$, by dividing the degree four polynomial function, $f(x)=3 x^{4}+13 x^{2}-30$, by the degree two polynomial function, $g(x)=3 x^{2}-5$, using long division. Connect the tabular representation with the symbolic representation.

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## Algebraic Reasoning

| $x$ | $f(x)$ | $g(x)$ | $h(x)$ <br> $f(x) \div g(x)$ | $h(x)$ |
| :---: | :---: | :---: | :---: | :---: |
| -2 | 70 | 7 | $70 \div 7$ | 10 |
| -1 | -14 | -2 | $-14 \div(-2)$ | 7 |
| 0 | -30 | -5 | $-30 \div(-5)$ | 6 |
| 1 | -14 | -2 | $-14 \div(-2)$ | 7 |
| 2 | 70 | 7 | $70 \div 7$ | 10 |
| 3 | 330 | 22 | $330 \div 22$ | 15 |
| 4 | 946 | 43 | $946 \div 43$ | 22 |


| $x$ | $h(x)$ <br> $x^{2}+6$ | $h(x)$ |
| :---: | :---: | :---: | :---: |
| -2 | $(-2)^{2}+6$ | 10 |
| -1 | $(-1)^{2}+6$ | 7 |
| 0 | $(0)^{2}+6$ | 6 |
| 1 | $(1)^{2}+6$ | 7 |
| 2 | $(2)^{2}+6$ | 10 |
| 3 | $(3)^{2}+6$ | 15 |
| 4 | $(4)^{2}+6$ | 22 |

Sample response:
Using a tabular representation, the evaluation of the independent values of $f(x)$ and $g(x)$ are divided to determine the dependent values of the quotient function, $h(x)=f(x) \div g(x)$.

Using a symbolic representation, the evaluation of the independent values of the quotient function, $h(x)=x^{2}+6$, determines the dependent values of the divided function, $h(x)$.

The quotient function can be represented by the quotient of the dependent values of $h(x)=f(x) \div g(x)$, which is equivalent to the evaluation of the independent values of the quotient function, $h(x)=x^{2}+6$.

Using a symbolic representation, solve the equation $3 x^{2}-5=0$ to determine if there are any $x$-values for which the quotient function, $h(x)$ is undefined.
$3 x^{2}-5=0$
$3 x^{2}=5$
$x^{2}=\frac{5}{3}$


The quotient function, $h(x)=\frac{3 x^{4}+13 x^{2}-30}{3 x^{2}-5}$, is not defined when $x=-\sqrt{\frac{5}{3}}$ and $x=\sqrt{\frac{5}{3}}$; therefore the quotient function is $h(x)=x^{2}+6, x \neq \pm \sqrt{\frac{5}{3}}$

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## Algebraic ReAsoning

- Ex:

A rectangular prism has a height represented by the function $h(x)=x+4$, and a volume represented by the function $V(x)=5 x^{3}+21 x^{2}+4 x$, where $x$ represents distance in inches. What function can be used to represent the area of the base, $B(x)$, of the prism?

## Sample response:

For a rectangular prism, the volume, $V$, area of the base, $B$, and height, $h$, are related by the formula $V=B h$.
To determine $B$, divide $V$ by $h$. Since each function is in terms of $x, B(x)=V(x) \div h(x)$.
Tabular
$V(x)=5 x^{3}+21 x^{2}+4 x$
$h(x)=x+4$
$B(x)=V(x) \div h(x)$

Use the quotient of the dependent values of $f(x)$ and $g(x)$ to determine the dependent values of the quotient function, $h(x)$.


$$
\begin{aligned}
& \text { Symbolic - Factoring } \\
& V(x)=5 x^{3}+21 x^{2}+4 x \\
& h(x)=x+4 \\
& B(x)=V(x) \div h(x)
\end{aligned}
$$

Use factoring methods to divide the degree three polynomial function, $f(x)$, by the degree one polynomial function, $g(x)$

$B(x)=x(5 x+1) \cdot \frac{(x+4)}{x+4}$
$B(x)=x(5 x+1) \cdot 1$
$B(x)=5 x^{2}+x$
The area of the base can be represented by the quotient function $B(x)=5 x^{2}+x$.

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## Algebraic Reasoning

| $x$ | $V(x)$ | $h(x)$ | $B(x)$ <br> $V(x) \div h(x)$ | $B(x)$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 4 | $0 \div 4$ | 0 |
| 1 | 30 | 5 | $30 \div 5$ | 6 |
| 2 | 132 | 6 | $132 \div 6$ | 22 |
| 3 | 336 | 7 | $336 \div 7$ | 48 |
| 4 | 672 | 8 | $672 \div 8$ | 84 |
| 5 | 1,170 | 9 | $1,170 \div 9$ | 130 |


| $x$ | $B(x)$ <br> $5 x^{2}+x$ | $B(x)$ |
| :---: | :---: | :---: |
| 0 | $5(0)^{2}+(0)$ | 0 |
| 1 | $5(1)^{2}+(1)$ | 6 |
| 2 | $5(2)^{2}+(2)$ | 22 |
| 3 | $5(3)^{2}+(3)$ | 48 |
| 4 | $5(4)^{2}+(4)$ | 84 |
| 5 | $5(5)^{2}+(5)$ | 130 |

## Sample response:

Using a tabular representation, the evaluation of the independent values of $V(x)$ and $h(x)$ are divided to determine the dependent values of the quotient function, $B(x)=V(x) \div h(x)$.

Using a symbolic representation, the evaluation of the independent values of the quotient function, $B(x)=5 x^{2}+x$, determines the dependent values of the divided function, $B(x)$.

The area of the base of the rectangular prism can be represented by the quotient of the dependent values of $B(x)=V(x) \div h(x)$, which is equivalent to the evaluation of the independent values of the quotient function, $B(x)=5 x^{2}+x$.

Since $x=-4$ results in division by 0 in the quotient function, $B(x)=\frac{5 x^{3}+21 x^{2}+4 x}{x+4}$, then $B(x)$ is not defined when $x=-4$. Further, since $x$ represents distance in inches, $x$ will only represent positive values or 0 . Within the context of the problem situation, the quotient function is represented by $B(x)=5 x^{2}+x, x \geq 0$.

Note(s):

- Grade Level(s):
- Algebra I determined the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend.
- Algebra I introduced regression methods for determining linear and quadratic functions.
- Algebraic Reasoning extends division of polynomials to degree three and degree four.
- Algebra Il will introduce synthetic division to divide higher degree polynomials.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- II.A. Algebraic Reasoning - Identifying expressions and equations


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## Algebraic ReAsoning

- II.A.1. Explain the difference between expressions and equations.
- II.B. Algebraic Reasoning - Manipulating expressions
- II.B.1. Recognize and use algebraic properties, concepts, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions.
- VI.B.2. Algebraically construct and analyze new functions.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.

Determine the linear factors of a polynomial function of degree two and of degree three when represented symbolically and tabularly and graphically where appropriate.

Determine
THE LINEAR FACTORS OF A POLYNOMIAL FUNCTION OF DEGREE TWO AND OF DEGREE THREE WHEN REPRESENTED SYMBOLICALLY AND TABULARLY AND GRAPHICALLY WHERE APPROPRIATE

Including, but not limited to:

- Linear factor - a degree one polynomial that is a factor of a polynomial expression or polynomial function
- Polynomial function - a relation that can be represented by a monomial or sum of monomials, not including variables in the denominator or under a radical, in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Determine linear factors of degree two and degree three polynomial functions
- Connections between $x$-intercepts (zeros) and linear factors
- If $x=c$ is a $x$-intercept (zero) of a polynomial function, then $(x-c)$ is a linear factor of the polynomial function.
- The $x$-intercepts of a polynomial function are equivalent to the $x$-intercepts of the polynomial function's linear factors.
- Symbolically
- Methods for determination of linear factors for degree two polynomial functions
- Box method
- Factor tables
- Ex:

Determine all linear factors of the polynomial function $f(x)=x^{2}-2 x-63$.
Box Method Factor Method

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## Algebraic Reasoning

1) Enter the squared term in the first box and the constant term in the last box.

| $x^{2}$ |  |
| :---: | :---: |
|  | -63 |

2) Determine the factors of the constant term, -63 , that sum to the coefficient of the middle term, -2 . Use the two factors to create $x$-terms to enter into the final two boxes.

Factors: $7+(-9)=-2$

| $x^{2}$ | $-9 x$ |
| :---: | :---: |
| $7 x$ | -63 |

3) Factor out the greatest common factor along each row, putting the factors to the left of each row, and factor out the greatest common factor vertically along each column, putting the factors on top of each row.

4) Write the linear factors using the terms to the left and the terms above the box, and check the linear factors by multiplying them back together to result in the original function.

Linear Factors: $(x+7)(x-9)$
Check
$f(x)=(x+7)(x-9)$
$f(x)=x^{2}-9 x+7 x-63$

1) Create a factor table to determine the factors of the last term, -63 , that sum to the coefficient of the middle term, -2 .

| Factors of -63 | Sum |
| :---: | :---: |
| $1,-63$ | -62 |
| $-1,63$ | 62 |
| $3,-21$ | -18 |
| $-3,21$ | 18 |
| $7,-9$ | -2 |
| $-7,9$ | 2 |

Factors: 7 and -9
2) Write the linear factors as $(x+$ factor $)(x+$ factor $)$ and simplify as necessary.
$(x+(7))(x+(-9))$
$(x+7)(x-9)$
3) Check the linear factors by multiplying them back together to result in the original function.
$f(x)=(x+7)(x-9)$
$f(x)=x^{2}-9 x+7 x-63$
$f(x)=x^{2}-2 x-63$
Linear Factors: $(x+7)$ and $(x-9)$

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## Algebraic REASONING

$f(x)=x^{2}-2 x-63$

- Ex:

Determine all linear factors of the function $f(x)=6 x^{2}+7 x-5$.
Box Method

1) Enter the squared term in the top left box and the constant term in the bottom right box.

| $6 x^{2}$ |  |
| :--- | :--- |
|  | -5 |

2) Determine the factors of the product of the coefficient of the square term, 6 , and the constant term, -5 , which will sum to the coefficient of the middle term, 7.

$$
\begin{gathered}
-3 \cdot 10=-30 \text { and }-3+10=7 \\
\text { Factors: }-3,10
\end{gathered}
$$

3) Use the two factors to create $x$-terms to enter into the final two boxes.

4) Factor out the greatest common factor along each row, putting the factors to the left of each row, and factor out the greatest common factor vertically along each column, putting the factors on top of each row.

5) Create a linear factor table using factors of the coefficient of $x^{2}, 6$, and the constant term, -5 . For each factor pair, use the distributive property to determine the product of the factor pairs that results in $6 x^{2}+7 x-5$.

| Factors | Product |
| :---: | :---: |
| $(2 x+5)(3 x-1)$ | $6 x^{2}-2 x+15 x-5$ |
| $6 x^{2}+13 x-5$ |  |
| $(2 x-5)(3 x+1)$ | $6 x^{2}+2 x-15 x-5$ |
|  | $6 x^{2}-13 x-5$ |
| $(-2 x+5)(-3 x-1)$ | $6 x^{2}+2 x-15 x-5$ |
| $6 x^{2}-13 x-5$ |  |
| $(-2 x-5)(-3 x+1)$ | $6 x^{2}-2 x+15 x-5$ |
|  | $6 x^{2}+13 x-5$ |
| $(2 x+1)(3 x-5)$ | $6 x^{2}-10 x+3 x-5$ |
| $6 x^{2}-7 x-5$ |  |
| $(x+5)(6 x-1)$ | $6 x^{2}-x+30 x-5$ |
|  | $6 x^{2}+29 x-5$ |
| $(6 x-5)(x+1)$ | $6 x^{2}+6 x-5 x-5$ |
|  | $6 x^{2}+x-5$ |
| $(-x+5)(-6 x-1)$ | $6 x^{2}+x-30 x-5$ |
|  | $6 x^{2}-29 x-5$ |
| $(-6 x-5)(-x+1)$ | $6 x^{2}-6 x+5 x-5$ |
|  | $6 x^{2}-x-5$ |
| $(2 x-1)(3 x+5)$ | $6 x^{2}+10 x-3 x-5$ |
|  | $6 x^{2}+7 x-5$ |

Linear Factors:

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- Methods for determination of linear factors for degree two and degree three polynomial functions
- Grouping
- Degree two polynomial functions
- Ex:



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## Algebraic Reasoning

1) Rewrite the middle term, 11x, as two terms with coefficients that combine to give the product of the leading coefficient, 4 , and the coefficient of the constant term, -3 .
$f(x)=4 x^{2}+11 x-3$
$f(x)=4 x^{2}+12 x-x-3$
2) Group the first two terms and the last two terms.
$f(x)=4 x^{2}+12 x-x-3$
$f(x)=\left(4 x^{2}+12 x\right)-(x+3)$
3) Factor out the greatest common factor from each grouped term, if possible.
$f(x)=\left(4 x^{2}+12 x\right)-(x+3)$
$f(x)=4 x(x+3)-(x+3)$
4) Factor out the common term, $(x+3)$ from the function on the left, or factor out the common term, $(4 x-1)$, from the function on the right.
$f(x)=4 x(x+3)-(x+3)$
$f(x)=(x+3)(4 x-1)$
Linear factors: $(x+3)$ and $(4 x-1)$
5) Check the linear factors by multiplying them back together to result in the original function.
$f(x)=(x+3)(4 x-1)$
$f(x)=x(4 x-1)+3(4 x-1)$
$f(x)=4 x^{2}-x+12 x-3$
$f(x)=4 x^{2}+11 x-3$
The linear factors of the function $f(x)=4 x^{2}+11 x-3$ are $(x+3)$ and $(4 x-1)$.

- Degree three polynomial functions
- Ex:

Determine all linear factors of the polynomial function $f(x)=x^{3}+3 x^{2}-4 x-12$ by grouping.

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## Algebraic Reasoning

1) Determine the greatest common factor of the first two terms and the greatest common factor of the last two terms.
$\operatorname{GCF}\left(x^{3}, 3 x^{2}\right)=x^{2}$
$\operatorname{GCF}(-4 x,-12)=-4$
2) Factor out the GCF, $x^{2}$, from the first two terms, and factor out the GCF, -4 , from the last two terms.
$f(x)=x^{3}+3 x^{2}-4 x-12$
$f(x)=x^{2}(x+3)-4(x+3)$
3) Factor out the common term, $(x+3)$, from the expression.
$f(x)=x^{2}(x+3)-4(x+3)$
$f(x)=(x+3)\left(x^{2}-4\right)$
4) Choose a factoring method to factor any remaining expressions, if necessary.
$f(x)=(x+3)\left(x^{2}-4\right)$
$f(x)=(x+3)(x+2)(x-2)$
5) Check the linear factors by multiplying them back together to result in the original function.
$f(x)=(x+3)(x+2)(x-2)$
$f(x)=\left(x^{2}+2 x+3 x+6\right)(x-2)$
$f(x)=\left(x^{2}+5 x+6\right)(x-2)$
$f(x)=x^{3}-2 x^{2}+5 x^{2}-10 x+6 x-12$
$f(x)=x^{3}+3 x^{2}-4 x-12$
The linear factors of the function $f(x)=x^{3}+3 x^{2}-4 x-12$ are $(x+3),(x+2)$, and $(x-2)$.

- Long division of degree two and degree three polynomial functions
- At least one factor given
- Synthetic division of degree two and degree three polynomial functions
- At least one linear factor given
- Ex:

Given the polynomial function, $f(x)=2 x^{2}-7 x-15$, and one of its linear factors, $(x-5)$, determine all remaining linear factors of the function.

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## Algebraic Reasoning



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## Algebraic Reasoning

Divide $7 x^{3}+2 x^{2}-89 x+60$ by its factor $(x+4)$.
$x + 4 \longdiv { 7 x ^ { 3 } + 2 x ^ { 2 } - 8 9 x + 6 0 }$


The factors of $f(x)=7 x^{3}+2 x^{2}-89 x+60$ are $(x+4)$ and $\left(7 x^{2}-26 x+15\right)$.

Determine the factors of $7 x^{2}-26 x+15$.

Box Method


Divide $7 x^{3}+2 x^{2}-89 x+60$ by its factor $(x+4)$.

| -4 | 7 | 2 | -89 | 60 |
| :--- | :--- | :--- | :--- | :--- |


|  | -28 | 104 | -60 |
| ---: | ---: | ---: | ---: |
| 7 | -26 | 15 | $\boxed{0}$ |

The $7,-26$, and 15 indicate a factor of $\left(7 x^{2}-26 x+15\right)$.
The factors of $f(x)=7 x^{3}+2 x^{2}-89 x+60$ are $(x+4)$ and $\left(7 x^{2}-26 x+15\right)$.

Determine the factors of $7 x^{2}-26 x+15$.
Grouping:
$7 x^{2}-26 x+15$
$7 x^{2}-21 x-5 x+15$
$7 x(x-3)-5(x-3)$
$(x-3)(7 x-5)$
The factors of $7 x^{2}-26 x+15$ are $(x-3)$ and $(7 x-5)$.
The linear factors of the function
$f(x)=7 x^{3}+2 x^{2}-89 x+60$ are $(x+4),(7 x-5)$, and $(x-3)$.

- Tabularly
- Locate $x$-intercepts (zeros) in a table of values ( $x$-values where $y=0$ )
- Write the $x$-intercepts (zeros) as linear factors
- Exact value of non-integral $x$-intercepts (zeros) may not be identifiable
- Graphically
- Locate $x$-intercepts (zeros) in a graph ( $x$-values where $y=0$ )
- Write the $x$-intercepts (zeros) as linear factors


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## Algebraic Reasoning

- Exact value of non-integral $x$-intercepts (zeros) may not be identifiable

Ex:
Given the polynomial function $f(x)=x^{2}+x-12$, determine the linear factors of $f(x)$ using symbolic, tabular, and graphical representations

Symbolic
Factor table:

| Factors of -12 | Sum |
| :---: | :---: |
| $1,-12$ | -11 |
| $-1,12$ | 11 |
| $2,-6$ | -4 |
| $-2,6$ | 4 |
| $3,-4$ | -1 |
| $-3,4$ | $\mathbf{1}$ |

Factors: $-3,4$
Use the factors to write the linear factors of $f(x)$.

Linear factors:
$(x+(-3))(x+(4))$
$(x-3)(x+4)$
Check:
$(x-3)(x+4)$
$x^{2}+4 x-3 x-12$
$x^{2}+x-12$
The linear factors of the function $f(x)=x^{2}+x-12$ are $(x-3)$ and $(x+4)$.

| $x$ | $f(x)$ |
| :---: | :---: |
| -5 | 8 |
| $x$ | $x$-intercept |
|  |  |
| -3 |  |
| -2 | -10 |
| -1 | -12 |
| 0 | -12 |
| 1 | -10 |
| 2 | -6 |
| 3 | 0 |
| 4 | 8 |

The table shows one $x$-intercept of -4 , located at the point $(-4,0)$. The table shows another $x$-intercept of 3 , located at the point $(3,0)$.

Use the $x$-intercepts to write the linear factors of $f(x)$.

$$
\begin{gathered}
x=-4 \text { and } x=3 \\
+4=0 \text { and } x-3=0 \\
(x+4)(x-3)=0
\end{gathered}
$$

The linear factors of the function
$f(x)=x^{2}+x-12$ are $(x+4)$ and $(x-3)$.

Graphical


The graph clearly shows one $x$-intercept of -4 , located at the point $(-4,0)$. The graph also clearly shows another $x$-intercept of 3 , located at the point $(3,0)$.

## Use the $x$-intercepts to write the linear factors

 of $f(x)$.$$
\begin{gathered}
x=-4 \text { and } x=3 \\
x+4=0 \text { and } x-3=0 \\
(x+4)(x-3)=0
\end{gathered}
$$

The linear factors of the function
$f(x)=x^{2}+x-12$ are $(x+4)$ and $(x-3)$.

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## Algebraic Reasoning

Grouping:
$x^{3}-5 x^{2}-4 x+20$
$x^{2}(x-5)-4(x-5)$
$(x-5)\left(x^{2}-4\right)$
Determine the factors of $x^{2}-4$.
Factor table:
Rewrite as $x^{2}+0 x-4$.

| Factors of -4 | Sum |
| :---: | :---: |
| $1,-4$ | -3 |
| $-1,4$ | 3 |
| $2,-2$ | 0 |
| $-2,2$ | 0 |

Factors: -2, 2

Use the factors -2 and 2 to write the linear factors of $x^{2}+$ $0 x-4$.

Linear factors:
$(x+(-2))(x+(2))$
$(x-2)(x+2)$

## Check:

$(x-5)(x-2)(x+2)$
$\left(x^{2}-2 x-5 x+10\right)(x+2)$
$\left(x^{2}-7 x+10\right)(x+2)$
$x^{3}+2 x^{2}-7 x^{2}-14 x+10 x+20$
$x^{3}-5 x^{2}-4 x+20$
The linear factors of the function
$f(x)=x^{3}-5 x^{2}-4 x+20$ are $(x-5),(x-2)$, and $(x+2)$.

| $x$ | $d(x)$ |
| :---: | :---: |
| -3 | -40 |
| $x$ | 0 |
|  |  |
| -1 | 18 |
| 0 | 20 |
| 1 | 12 |
| $x$ |  |
|  | 0 |
|  | -10 |
| 4 | -12 |

The table shows one $x$-intercept of -2 , located at the point $(-2,0)$. The table shows a second $x$-intercept of 2 , located at the point $(2,0)$. The table shows a third $x$-intercept of 5 , located at the point $(5,0)$

Use the $x$-intercepts to write the linear factors of $f(x)$.

$$
x=-2 \text { and } x=2 \text { and } x=5
$$

$$
x+2=0 \text { and } x-2=0 \text { and } x-5=0
$$

$$
(x+2)(x-2)(x-5)=0
$$

The linear factors of the function
$f(x)=x^{3}-5 x^{2}-4 x+20$ are $(x+2),(x-2)$ and $(x-5)$.

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## Algebraic Reasoning

- Ex:

Given the function $m(x)=6 x^{3}-11 x^{2}-19 x-6$ and one of its linear factors $(x-3)$, determine all linear factors of $m(x)$ using symbolic, tabular, and graphical representations.

| Symbolic | Tabular | Graphical |
| :---: | :---: | :---: |
| Polynomial long division: <br> The factors of $6 x^{3}-11 x^{2}-19 x$ -6 are $(x-3)$ and $\left(6 x^{2}+7 x+2\right)$. <br> Determine the factors of $6 x^{2}+7 x+2$ <br> Box Method: <br> The factors of $6 x^{2}+7 x+2$ are $(3 x+2)$ and $(2 x+1)$. <br> The factors of $\begin{aligned} & 6 x^{3}-11 x^{2}-19 x-6 \text { are } \\ & (x-3),(3 x+2), \text { and }(2 x+1) \end{aligned}$ | $x$ $m(x)$ <br> -2 -60 <br> -1 -4 <br> 0 -6 <br> 1 -30 <br> 2 -40 <br> 3 0 <br> 4 126 <br> 5 374 <br> $x$-intercept(s) ?? <br> The table shows a $x$-intercept of 3 , located at the point $(3,0)$. The $x$-value(s) of another possible $x$-intercept(s) is not readily identifiable in the table. There is no change of sign in the $y$-values in the table (like there is on either side of the $x$-value, $x=3$ ), so the tabular representation gives no indication of other possible $x$-intercepts. In fact, there are two $x$-intercepts between the $x$-values -1 and 0 . The sign of the $y$-values changes from negative to positive on either side of one of the $x$-intercepts, and then back from positive to negative on either side of the other $x$-intercept, which is why there is no indication of a sign change in the $y$-values between the $x$-values -1 and 0 . <br> Since the exact values of the other two |  <br> The graph clearly shows a $x$-intercept of 3 , located at the point $(3,0)$. The $x$-value(s) of another possible $x$-intercept(s) is not readily identifiable in the graph. The graph appears to show one or two $x$-intercepts between the $x$ values -1 and 0 . The value of either potential $x$-intercept is not clear. In fact, there are two $x$ intercepts between the $x$-values -1 and 0 . From the graph alone, the exact value of these $x$-intercepts is not clear. <br> Since the exact value of the other two $x$-intercepts cannot be determined from the displayed graph, use technology to find the remaining zeros of the graph. The determined |

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## Algebraic Reasoning

Matrix A has 2 rows. The first row, $r_{1}$, has elements $7,-1.5$, and 1. The
second row, $r_{2}$, has elements 3,0 , and -7 .

- Column - vertical arrangement of elements in an array
- Ex:

Matrix $A$
$c_{1}$
$c_{2}$
$c_{3}$
$\left[\begin{array}{ccc}7 & -1.5 & 1 \\ 3 & 0 & -7\end{array}\right]$

Matrix $A$ has 3 columns. The first column, $c_{1}$, has elements 7 and 3 . The second column, $c_{2}$, has elements of -1.5 and 0 . The third column, $c_{3}$, has elements 1 and -7 .

- Matrices are represented with a capital, italicized letter and identified by their dimensions, rows $\times$ columns (e.g., matrix $A$ that has 2 rows and 3 columns is described as, "matrix $A$ is a $2 \times 3$ matrix.")
- Ex:

- Operations with matrices
- Corresponding elements - elements of two matrices that are in the same row and same column position in each matrix
- Ex:


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## Algebraic Reasoning

$a_{1,2}$ and $b_{1,2}$ are corresponding elements of matrix $A$ and matrix $B$ because each element is located in the first row, second column of each matrix.

- Addition of matrices
- Addend matrix - one of the matrices being added
- Sum matrix - a result matrix representing the sum of matrices
- The sum matrix has the same dimensions as the addend matrices.
- The sum matrix elements are determined by adding the corresponding elements from each addend matrix.
addend matrix + addend matrix $=$ sum matrix
$\left[\begin{array}{cc}3 \times 2 \\ a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2}\end{array}\right]+\left[\begin{array}{cc}3 \times 2 \\ b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \\ b_{3,1} & b_{3,2}\end{array}\right]=\left[\begin{array}{ll}a_{1,1}+b_{1,1} & a_{1,2}+b_{1,2} \\ a_{2,1}+b_{2,1} & a_{2,2}+b_{2,2} \\ a_{3,1}+b_{3,1} & a_{3,2}+b_{3,2}\end{array}\right]$
- Addition of matrices is possible only when the matrices have the same dimensions (e.g., the number of rows and columns of the first addend matrix are the same as the number of rows and columns in the second addend matrix).
- Ex:
Given $A=\left[\begin{array}{lll}23 & 83 & 62 \\ 31 & 92 & 77\end{array}\right]$ and $B=\left[\begin{array}{lll}12 & 58 & 37 \\ 17 & 65 & 44\end{array}\right]$, find $A+B$.

$$
\begin{aligned}
& A+B \\
& {\left[\begin{array}{ccc}
23 & 83 & 62 \\
31 & 92 & 77
\end{array}\right]+\left[\begin{array}{ccc}
2 \times 3 & 58 & 37 \\
17 & 65 & 44
\end{array}\right]=\left[\begin{array}{lll}
23+12 & 83+58 & 62+37 \\
31+17 & 92+65 & 77+44
\end{array}\right]=\left[\begin{array}{ccc}
35 & 141 & 99 \\
48 & 157 & 121
\end{array}\right]}
\end{aligned}
$$

- Ex:



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- Properties of Addition with Matrices


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## Algebraic Reasoning

- Commutative Property of Addition with Matrices - if the order of the addend matrices is changed, the sum matrix will remain the same (e.g., matrix $A+$ matrix $B=$ matrix $C$; therefore, matrix $B+$ matrix $A=$ matrix $C$ )
$A$
$\left[\begin{array}{ll}a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2}\end{array}\right]+\left[\begin{array}{ll}b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \\ b_{3,1} & b_{3,2}\end{array}\right]=\left[\begin{array}{ll}b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \\ b_{3,1} & b_{3,2}\end{array}\right]+\left[\begin{array}{ll}a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2}\end{array}\right]$
- Ex:

Given $A=\left[\begin{array}{rr}2 & 6 \\ -4 & 7 \\ 1 & 3\end{array}\right]$ and $B=\left[\begin{array}{rr}1 & 2 \\ 8 & -1 \\ -6 & 5\end{array}\right]$,
find $A+B$ and $B+A$ to determine if the commutative property holds true for the addition of matrices with the same dimensions.


Since the sum matrices are the same, the commutative property of addition for matrices holds true.

- Associative Property of Addition with Matrices - if three or more addend matrices are added, they can be grouped in any order and the sum will remain the same (e.g., (matrix $A+$ matrix $B$ ) + matrix $C=$ matrix $A+($ matrix $B+$ matrix $C)$ )


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## Algebraic REASONING

$$
\left(\left[\begin{array}{ll}
a_{1,1} & a_{1,2} \\
a_{2,1} & a_{2,2} \\
a_{3,1} & a_{3,2}
\end{array}\right]+\left[\begin{array}{ll}
b_{1,1} & b_{1,2} \\
b_{2,1} & b_{2,2} \\
b_{3,1} & b_{3,2}
\end{array}\right]\right)+\left[\begin{array}{ll}
c_{1,1} & c_{1,2} \\
c_{2,1} & c_{2,2} \\
c_{3,1} & c_{3,2}
\end{array}\right]=\left[\begin{array}{ll}
a_{1,1} & a_{1,2} \\
a_{2,1} & a_{2,2} \\
a_{3,1} & a_{3,2}
\end{array}\right]+\left(\left[\begin{array}{ll}
b_{1,1} & b_{1,2} \\
b_{2,1} & b_{2,2} \\
b_{3,1} & b_{3,2}
\end{array}\right]+\left[\begin{array}{l}
c_{1,1} \\
c_{2,1} \\
c_{1,2} \\
c_{2,2} \\
c_{3,1}
\end{array}\right]\right)
$$

- Ex:

Given $A=\left[\begin{array}{rr}2 & 6 \\ -4 & 7 \\ 1 & 3\end{array}\right], B=\left[\begin{array}{rr}1 & 2 \\ 8 & -1 \\ -6 & 5\end{array}\right]$, and $C=\left[\begin{array}{rr}-5 & -8 \\ 10 & -3 \\ 2 & 11\end{array}\right]$,
use the order of operations to find $A+(B+C)$ and $(A+B)+C$ to determine if the associative property holds true for the addition of matrices.
$A+(B+C)$

$$
\begin{aligned}
& {\left[\begin{array}{rr}
2 & 6 \\
-4 & 7 \\
1 & 3
\end{array}\right]+\left(\left[\begin{array}{rr}
1 & 2 \\
8 & -1 \\
-6 & 5
\end{array}\right]+\left[\begin{array}{rr}
-5 & -8 \\
10 & -3 \\
2 & 11
\end{array}\right]\right)=\left[\begin{array}{rr}
2 & 6 \\
-4 & 7 \\
1 & 3
\end{array}\right]+\left[\begin{array}{cc}
-4 & -6 \\
18 & -4 \\
-4 & 16
\end{array}\right]=\left[\begin{array}{cc}
-2 & 0 \\
14 & 3 \\
-3 & 19
\end{array}\right]} \\
& \left(\left[\begin{array}{rr}
2 & 6 \\
-4 & 7 \\
1 & 3
\end{array}\right]+\left[\begin{array}{rr}
1 & 2 \\
8 & -1 \\
-6 & 5
\end{array}\right]\right)+\left[\begin{array}{rr}
-5 & -8 \\
10 & -3 \\
2 & 11
\end{array}\right]=\left[\begin{array}{cc}
3 & 8 \\
4 & 6 \\
-5 & 8
\end{array}\right]+\left[\begin{array}{cc}
-5 & -8 \\
10 & -3 \\
2 & 11
\end{array}\right]=\left[\begin{array}{cc}
-2 & 0 \\
14 & 3 \\
-3 & 19
\end{array}\right]
\end{aligned}
$$

Since the sum matrices are the same, the associative property of addition of matrices holds true.

- Subtraction of matrices
- Minuend matrix - a matrix from which another matrix is being subtracted
- Subtrahend matrix - a matrix that is being subtracted from another matrix
- Difference matrix - a result matrix representing the difference of matrices
- Difference matrix elements are determined by subtracting corresponding elements of the subtrahend matrix from the minuend matrix.


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## Algebraic REASONING

minuend matrix - subtrahend matrix $=$ difference matrix

$$
\left[\begin{array}{cc}
3 \times 2 \\
b_{1,1} & b_{1,2} \\
b_{2,1} & b_{2,2} \\
b_{3,1} & b_{3,2}
\end{array}\right]-\left[\begin{array}{cc}
3 \times 2 \\
a_{1,1} & a_{1,2} \\
a_{2,1} & a_{2,2} \\
a_{3,1} & a_{3,2}
\end{array}\right]=\left[\begin{array}{cc}
b_{1,1}-a_{1,1} & b_{1,2}-a_{1,2} \\
b_{2,1}-a_{2,1} & b_{2,2}-a_{2,2} \\
b_{3,1}-a_{3,1} & b_{3,2}-a_{3,2}
\end{array}\right]
$$

- Subtraction of matrices is possible only when the matrices have the same dimensions (e.g., number of rows and columns of the minuend matrix is the same as the number of rows and columns in the subtrahend matrix).
- Subtraction of matrices requires all matrices to have the same dimensions.
- Ex:

- Ex:


$$
A-B
$$

$$
\left[\begin{array}{cc}
2 \times 2 \\
23 & 83 \\
31 & 92
\end{array}\right]-\left[\begin{array}{cc}
2 \times 3 \\
12 & 58 \\
17 & 65
\end{array}\right.
$$



These matrices cannot be subtracted because they do not have the same dimensions

- Real-world applications
- Ex:


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## Algebraic Reasoning

The local zoo recorded the number of visitors in a particular exhibit. The total number of visitors for
Friday - Sunday for Week A and Week B are shown in the tables.

| Week A |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Friday | Saturday | Sunday |
| Adults | 23 | 83 | 62 |
| Children | 31 | 92 | 77 |


| Week B |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Friday | Saturday | Sunday |
| Adults | 12 | 58 | 37 |
| Children | 17 | 65 | 44 |

Use matrices to represent Week A and Week B. Use the matrices to determine how many more adults and children visited the exhibit on Week A than on Week B on each day of the week.

Write matrix $A=\left[\begin{array}{lll}23 & 83 & 62 \\ 31 & 92 & 77\end{array}\right]$ and matrix $B=\left[\begin{array}{lll}12 & 58 & 37 \\ 17 & 65 & 44\end{array}\right]$. $A-B$


|  | Friday | Saturday | Sunday |
| :--- | :---: | :---: | :---: |
| Adults | 11 | 25 | 25 |
| Children | 14 | 27 | 33 |

- Properties of Subtraction with Matrices
- Subtraction is not commutative because if the subtrahend and minuend matrices are interchanged, the difference matrices are not the same (e.g., matrix $A$ - matrix $B \neq$ matrix $B$ - matrix $A$ ).
- Ex:


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## Algebraic REASONING

Given $A=\left[\begin{array}{rr}2 & 6 \\ -4 & 7 \\ 1 & 3\end{array}\right]$ and $B=\left[\begin{array}{rr}1 & 2 \\ 8 & -1 \\ -6 & 5\end{array}\right]$,
find $A-B$ and $B-A$ to determine if the commutative property holds true for the subtraction of matrices.
$A-B$

| $\left[\begin{array}{rr}2 & 6 \\ -4 & 7 \\ 1 & 3\end{array}\right]-\left[\begin{array}{rr}1 & 2 \\ 8 & -1 \\ -6 & 5\end{array}\right]=\left[\begin{array}{cc}2-1 & 6-2 \\ -4-8 & 7-(-1) \\ 1-(-6) & 3-5\end{array}\right]=\left[\begin{array}{cc}1 & 4 \\ -12 & 8 \\ 7 & -2\end{array}\right]$ |  |
| :--- | :--- |
|  | $\left[\begin{array}{rr}1 & 2 \\ 8 & -1 \\ -6 & 5\end{array}\right]-\left[\begin{array}{rr}2 & 6 \\ -4 & 7 \\ 1 & 3\end{array}\right]=\left[\begin{array}{cc}1-2 & 2-6 \\ 8-(-4) & -1-7 \\ -6-1 & 5-3\end{array}\right]=\left[\begin{array}{cc}-1 & -4 \\ 12 & -8 \\ -7 & 2\end{array}\right]$ |

Since the difference matrices are not the same, subtraction of matrices is not commutative.

- Subtraction of matrices is not associative because if the grouping of the matrices is changed, the difference matrices are not the same.
- Ex:

find $A-(B-C)$ and $(A-B)-C$ to determine if the associative property holds true for the subtraction of matrices.
$A-(B-C)$


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## Algebraic ReAsoning

$$
\begin{aligned}
& {\left[\begin{array}{rr}
2 & 6 \\
-4 & 7 \\
1 & 3
\end{array}\right]-\left(\left[\begin{array}{rr}
1 & 2 \\
8 & -1 \\
-6 & 5
\end{array}\right]-\left[\begin{array}{rr}
-5 & -8 \\
10 & -3 \\
2 & 11
\end{array}\right]\right)=\left[\begin{array}{rr}
2 & 6 \\
-4 & 7 \\
1 & 3
\end{array}\right]-\left[\begin{array}{cc}
6 & 10 \\
-2 & 2 \\
-8 & -6
\end{array}\right]=\left[\begin{array}{cc}
-4 & -4 \\
-2 & 5 \\
9 & 9
\end{array}\right]} \\
& (A-B)-C \\
& \left(\left[\begin{array}{rr}
2 & 6 \\
-4 & 7 \\
1 & 3
\end{array}\right]-\left[\begin{array}{rr}
1 & 2 \\
8 & -1 \\
-6 & 5
\end{array}\right]\right)-\left[\begin{array}{cc}
-5 & -8 \\
10 & -3 \\
2 & 11
\end{array}\right]=\left[\begin{array}{cc}
1 & 4 \\
-12 & 8 \\
7 & -2
\end{array}\right]-\left[\begin{array}{cc}
-5 & -8 \\
10 & -3 \\
2 & 11
\end{array}\right]=\left[\begin{array}{cc}
6 & 12 \\
-22 & 11 \\
5 & -13
\end{array}\right]
\end{aligned}
$$

Since the difference matrices are not the same, subtraction of matrices is not associative.

- Addition and subtraction of matrices
- Addition and subtraction of matrices requires all matrices to have the same dimensions.
- As with the order of real number operations, addition and subtraction of matrices are performed from left to right.
- Ex:

$$
\text { Given } A=\left[\begin{array}{rr}
2 & 6 \\
-4 & 7 \\
1 & 3
\end{array}\right], B=\left[\begin{array}{rr}
1 & 2 \\
8 & -1 \\
-6 & 5
\end{array}\right] \text {, and } C=\left[\begin{array}{rr}
-5 & -8 \\
10 & -3 \\
2 & 11
\end{array}\right] \text {, find } A-B+C .
$$

$$
A-B+C
$$

$$
\left[\begin{array}{cc}
2 & 6 \\
-4 & 7 \\
1 & 3
\end{array}\right]-\left[\begin{array}{rr}
1 & 2 \\
8 & -1 \\
-6 & 5
\end{array}\right]+\left[\begin{array}{cc}
-5 & -8 \\
10 & -3 \\
2 & 11
\end{array}\right]=\left[\begin{array}{cc}
1 & 4 \\
-12 & 8 \\
7 & -2
\end{array}\right]+\left[\begin{array}{cc}
-5 & -8 \\
10 & -3 \\
2 & 11
\end{array}\right]=\left[\begin{array}{cc}
-4 & -4 \\
-2 & 5 \\
9 & 9
\end{array}\right]
$$

Note(s):

- Grade Level(s):
- Elementary used arrays to represent mathematical structures, including multiplication and division facts.


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## Algebraic Reasoning

- Algebra I used properties of algebra to add and subtract first and second degree polynomials.
- Algebraic Reasoning uses arrays to represent and structure mathematical ideas, including data and components of systems of equations (coefficients and variables).
- Algebraic Reasoning introduces matrices and addition and subtraction of matrices.
- Algebra Il will use matrices as a tool to solve systems of equations.
- Algebra Il will use properties of algebra to add and subtract rational expressions.
- Advanced Quantitative Reasoning will use addition and subtraction of matrices to solve problems involving arrays of data.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- I.A. Numeric Reasoning - Number representations and operations
- I.A.2. Perform computations with rational and irrational numbers.
- II.A. Algebraic Reasoning - Identifying expressions and equations
- II.A.1. Explain the difference between expressions and equations.
- II.B. Algebraic Reasoning - Manipulating expressions
- II.B.1. Recognize and use algebraic properties, concepts, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).


## Multiply matrices.

Multiply

## MATRICES

Including, but not limited to:

- Matrix - a rectangular array of data elements
- Matrices are represented by a capitalized, italicized letter.
- Dimensions are rows $\times$ columns and represented as $r \times c$.
- Element - one of the data entries in a matrix
- Elements are represented by a lower case, italicized letter of the matrix name with subscript indexes.
- Index - a subscript used to indicate the location of an element in terms of its row and column position in a matrix (e.g., $a_{2,3}$ indicates the element located in the $2^{\text {nd }}$ row and $3^{\text {rdd }}$ column in matrix $A$.)
- Ex:


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## Algebraic Reasoning

- Structure of a matrix
- Row - horizontal arrangement of elements in an array
- Ex:

Matrix $A$


Matrix A has 2 rows. The first row, $r_{1}$, has elements $7,-1.5$, and 1. The second row, $r_{2}$, has elements 3,0 , and -7 .

- Column - vertical arrangement of elements in an array
- Ex:


## Matrix $A$



Matrix $A$ has 3 columns. The first column, $c_{1}$, has elements 7 and 3 . The second column, $c_{2}$, has elements -1.5 and 0 . The third column, $c_{3}$, has elements 1 and -7 .

- Matrices are represented with a capital, italicized letter and identified by their dimensions, rows $\times$ columns (e.g., Matrix $A$ that has 2 rows and 3 columns is described as, "Matrix $A$ is a $2 \times 3$ matrix.")
- Ex:


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## Algebraic Reasoning

Matrix $A$ has 2 rows and 3 columns. Matrix $A$ is a $2 \times 3$ matrix.

- Operations with matrices
- Multiplication of matrices
- Factor matrix - one of the two matrices being multiplied
- Product matrix - a result matrix representing the product of two matrices
- The dimensions of the product matrix are the number of rows of the first factor matrix by the number of columns of the second factor matrix.
- Multiplication of matrices is possible only when the number of columns of the first factor matrix is the same as the number of rows in the second factor matrix.
- Product matrix elements are determined by taking the sums of the products of paired entries (row of first factor matrix $\times$ column of second factor matrix) of matrices that can be multiplied based on their dimensions.


$$
\underline{2} \times 3 \quad 3 \times \underline{1}
$$

$$
\underline{2} \times \underline{1}
$$

$$
\left[\begin{array}{lll}
a & b & c \\
d & e & f
\end{array}\right]\left[\begin{array}{l}
h \\
j \\
k
\end{array}\right]=\left[\begin{array}{l}
a h+b j+c k \\
d h+e j+f k
\end{array}\right]
$$

The number of columns in the first factor matrix is the same as the number rows in the second factor matrix (both are 3).

The dimensions of the product matrix are the number of rows of the first factor matrix by the number of columns of the second factor matrix $(2 \times 1)$.

Sum of each product of elements in row $1 \times$ column 1 .
Sum of each product of elements in row $2 \times$ column 1 .

- Ex:


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## Algebraic Reasoning

$$
\left[\begin{array}{cc}
6 & 1 \\
9 & 7 \\
-2 & 5
\end{array}\right]\left[\begin{array}{cc}
2 & -2 \\
5 & 6
\end{array}\right]=\left[\begin{array}{cc}
(6 \times 2)+(1 \times 5) & (6 \times-2)+(1 \times 6) \\
(9 \times 2)+(7 \times 5) & (9 \times-2)+(7 \times 6) \\
(-2 \times 2)+(5 \times 5) & (-2 \times-2)+(5 \times 6)
\end{array}\right]=\left[\begin{array}{cc}
17 & -6 \\
53 & 24 \\
21 & 34
\end{array}\right]
$$

- Real-world applications
- Ex:

Thuy and Yesenia work at a local movie theater. They both sold tickets in the ticket booth on Tuesday. Which employee will have the higher receipt total at the end of their shifts?

|  | Number of <br> Child Tickets <br> Sold | Number of <br> Student Tickets <br> Sold | Number of <br> Adult Tickets <br> Sold | Number of <br> Senior Tickets <br> Sold |
| :---: | :---: | :---: | :---: | :---: |
| Thuy | 60 | 48 | 144 | 30 |
| Yesenia | 102 | 27 | 108 | 60 |


|  | Cost per Ticket <br> (dollars) |
| :---: | :---: |
| Child | $\$ 7.25$ |
| Student | $\$ 10.00$ |
| Adult | $\$ 12.50$ |
| Senior | $\$ 8.00$ |

Write matrix $T$ to show the number of tickets each employee sold.

$$
T=\left[\begin{array}{cccc}
60 & 48 & 144 & 30 \\
102 & 27 & 108 & 60
\end{array}\right]
$$

Write matrix $C$ to show the cost of each type of ticket.

$$
C=\left[\begin{array}{c}
7.25 \\
10.00 \\
12.50 \\
8.00
\end{array}\right]
$$

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## Algebraic REASONING

Multiply matrix $T$ by matrix $C$.


Thuy sold $\$ 2955$ worth of tickets and Yesenia sold $\$ 2839.50$ worth of tickets, Thuy had a higher receipt total.

- Properties of Operations
- Matrix multiplication is not commutative.
- Two matrices that can be multiplied in one orientation may not be able to be multiplied if the order of the matrices is commuted.
- Ex:

$$
\text { Given } A=\left[\begin{array}{ccc}
18 & 42 & 7 \\
23 & 37 & 10
\end{array}\right] \text { and } B=\left[\begin{array}{l}
1 \\
2 \\
3
\end{array}\right]
$$

find $A B$ and $B A$ to determine if the commutative property holds true for multiplication of matrices.

| $A B$ $\left[\begin{array}{ccc} 18 & 42 & 7 \\ 23 & 37 & 10 \end{array}\right]\left[\begin{array}{l} 1 \\ 2 \\ 3 \end{array}\right]$ | $\begin{array}{r} B A \\ \\ \\ \\ 3 \times \underline{1} \quad\left[\begin{array}{l} 1 \\ 2 \\ 3 \end{array}\right]\left[\begin{array}{ccc} 18 & 42 & 7 \\ 23 & 37 & 10 \end{array}\right] \end{array}$ |
| :---: | :---: |
| Matrix dimensions indicate matrices can be multiplied since the number of columns in matrix $A$ equals the number of rows in Matrix $B$. | Interchanged matrices' dimensions indicate matrices cannot be multiplied since the number of columns in matrix $B$ does not equal the number of rows in matrix $A$. |

Since the dimensions of the interchanged matrices indicate multiplication is not possible, matrix multiplication is not commutative.

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## Algebraic REASONING

- Matrices that can be multiplied may create interchanged matrices that can also be multiplied, but the product matrices are not the same. Therefore, matrix multiplication is not commutative.
- Ex:



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## Algebraic Reasoning

Since the product of $A B$ and the product of $B A$ do not result in matrices with the same dimensions and the same product matrices, the matrix multiplication is not commutative.

- Ex:

Given $A=\left[\begin{array}{ll}8 & 4 \\ 3 & 7\end{array}\right]$ and $B=\left[\begin{array}{ll}1 & 2 \\ 2 & 3\end{array}\right]$,
find $A B$ and $B A$ to determine if the commutative property holds true for multiplication of matrices.

| Matrices | Matrices |
| :---: | :---: |
| $\begin{array}{r} A B \quad\left[\begin{array}{ll} 8 & 4 \\ 3 & 7 \end{array}\right]\left[\begin{array}{ll} 1 & 2 \\ 2 & 3 \end{array}\right] \\ \\ 2 \times \underline{\mathbf{2}} \quad \underline{\mathbf{2}} \times 2 \end{array}$ | $\begin{aligned} & \text { BA } \\ & 2 \times \underline{\mathbf{2}} \quad\left[\begin{array}{ll} 1 & 2 \\ 2 & 3 \end{array}\right]\left[\begin{array}{ll} 8 & 4 \\ 3 & 7 \end{array}\right] \\ & \end{aligned}$ |
| Matrix dimensions indicate matrices can be multiplied since the number of columns in matrix $A$ equals the number of rows in matrix $B$. | Interchanged matrices' dimensions indicate matrices can be multiplied since the number of columns in matrix $B$ equals the number of rows in matrix $A$. |
| $\begin{gathered} \text { AB } \\ {\left[\begin{array}{ll} 8 & 4 \\ 3 & 7 \end{array}\right]\left[\begin{array}{ll} 1 & 2 \\ 2 & 3 \end{array}\right]} \\ 2 \times \underline{\underline{2}} \\ \underline{\mathbf{2}} \times 2 \\ {\left[\begin{array}{ll} 8 & 4 \\ 3 & 7 \end{array}\right]\left[\begin{array}{ll} 1 & 2 \\ 2 & 3 \end{array}\right]=\left[\begin{array}{ll} 16 & 28 \\ 17 & 27 \end{array}\right]} \\ 2 \times \underline{\mathbf{2}} \\ \underline{\mathbf{2}} \times 2= \\ 2 \times 2 \end{gathered}$ | $\begin{gathered} \text { BA } \\ \left.2 \times \underline{\underline{\mathbf{2}}} \begin{array}{ll} 1 & 2 \\ 2 & 3 \end{array}\right]\left[\begin{array}{ll} 8 & 4 \\ 3 & 7 \end{array}\right] \\ \\ {\left[\begin{array}{ll} 1 & 2 \\ 2 & 3 \end{array}\right]\left[\begin{array}{ll} 8 & 4 \\ 3 & 7 \end{array}\right]=\left[\begin{array}{ll} 14 & 18 \\ 25 & 29 \end{array}\right]} \\ 2 \times \underline{\mathbf{2}} \quad \underline{\mathbf{2}} \times 2=2 \times 2 \end{gathered}$ |

- Order of matrices being multiplied matters
- Left-multiplication when a matrix is multiplied by a matrix to its left
- Right-multiplication when a matrix is multiplied by a matrix to its right.


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## Algebraic ReAsoning

- Ex:

Let Matrix $C$ be $\left[\begin{array}{ccc}2 & 5 & -2 \\ 1 & 7 & 10\end{array}\right]$ and Matrix $D$ be $\left[\begin{array}{cc}6 & 1 \\ 9 & 7 \\ -2 & 5\end{array}\right]$
Left-multiply matrix $D$ by matrix $C$.
$C D=\left[\begin{array}{ccc}2 & 5 & -2 \\ 1 & 7 & 10\end{array}\right]\left[\begin{array}{cc}6 & 1 \\ 9 & 7 \\ -2 & 5\end{array}\right]=\left[\begin{array}{cc}61 & 27 \\ 49 & 100\end{array}\right]$
Right-multiply matrix $D$ by matrix $C$.

Left-multiplication generates a $2 \times 2$ matrix and right-multiplication generates a $3 \times 3$ matrix.

- Associative Property of Multiplication of Matrices - if three or more factor matrices are multiplied, they can be grouped in any order, and the product will remain the same (e.g., (matrix $A \times$ matrix $B$ ) $\times$ matrix $C=$ matrix $A \times($ matrix $B \times$ matrix $C$ )
- Regrouping of factor matrices that can be multiplied due to their dimensions results in the same product matrix.

- Ex:

find $A(B C)$ and $(A B) C$ to determine if the associative property of multiplication holds true for matrices.
$A(B C)$

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Since the products matrices are the same, the associative property of multiplication holds true for matrices.

- Distributive Property of Matrix Multiplication Over Addition - if multiplying a matrix by the sum of matrices, the product will be the same as multiplying the matrix by each addend matrix and then adding the product matrices together
$\left[\begin{array}{lll}a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3}\end{array}\right]\left(\left[\begin{array}{l}b_{1,1} \\ b_{2,1} \\ b_{3,1}\end{array}\right]+\left[\begin{array}{l}c_{1,1} \\ c_{2,1} \\ c_{3,1}\end{array}\right]\right)=\left(\left[\begin{array}{lll}a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3}\end{array}\right]\left[\begin{array}{l}b_{1,1} \\ b_{2,1} \\ b_{3,1}\end{array}\right]\right)+\left(\left[\begin{array}{lll}a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3}\end{array}\left[\begin{array}{l}c_{1,1} \\ c_{2,1} \\ c_{3,1}\end{array}\right]\right)\right.$
- Ex:

find $A(B+C)$ and $A B+A C$ to determine if the distributive property of matrix multiplication over addition holds true.
$A(B+C)$


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## Algebraic Reasoning

$$
\begin{aligned}
& {\left[\begin{array}{ccc}
18 & 42 & 7 \\
23 & 37 & 10
\end{array}\right]\left(\left[\begin{array}{c}
2 \\
-3 \\
1
\end{array}\right]+\left[\begin{array}{c}
-3 \\
0 \\
2
\end{array}\right]\right)=\left[\begin{array}{lll}
18 & 42 & 7 \\
23 & 37 & 10
\end{array}\right]\left[\begin{array}{c}
-1 \\
-3 \\
3
\end{array}\right]=\left[\begin{array}{c}
-123 \\
-104
\end{array}\right]} \\
& A B+B C \\
& {\left[\begin{array}{ccc}
18 & 42 & 7 \\
23 & 37 & 10
\end{array}\right]\left(\left[\begin{array}{c}
2 \\
-3 \\
1
\end{array}\right]+\left[\begin{array}{c}
-3 \\
0 \\
2
\end{array}\right]\right)=\left[\begin{array}{ll}
18 & 42 \\
23 & 37 \\
10
\end{array}\right]\left[\begin{array}{c}
2 \\
-3 \\
1
\end{array}\right]+\left[\begin{array}{ccc}
18 & 42 & 7 \\
23 & 37 & 10
\end{array}\right]\left[\begin{array}{c}
-3 \\
0 \\
2
\end{array}\right]} \\
& =\left[\begin{array}{c}
-83 \\
-55
\end{array}\right]+\left[\begin{array}{c}
-123 \\
-104
\end{array}\right]
\end{aligned}
$$

Since the product matrices are the same, the distributive property matrix multiplication over addition holds true.

Note(s):

- Grade Level(s):
- Algebra I used properties of algebra to multiply polynomials.
- Algebraic Reasoning uses properties of algebra to multiply matrices.
- Algebraic Reasoning uses matrix multiplication to solve systems of linear equations in 2 or 3 variables.
- Algebra II will use properties of algebra to multiply rational expressions.
- Algebra II will use matrix multiplication to multiply an inverse matrix by the constant matrix.
- Advanced Quantitative Reasoning will use matrix multiplication to solve problems involving arrays of data.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- I.A. Numeric Reasoning - Number representations and operations
- I.A.2. Perform computations with rational and irrational numbers.
- II.A. Algebraic Reasoning - Identifying expressions and equations
- II.A.1. Explain the difference between expressions and equations.
- II.B. Algebraic Reasoning - Manipulating expressions


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## Algebraic Reasoning

- II.B.1. Recognize and use algebraic properties, concepts, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).


## AR.5C Multiply matrices by a scalar.

Multiply

## MATRICES BY A SCALAR

Including, but not limited to:

- Matrix - a rectangular array of data elements
- Matrices are represented by a capitalized, italicized letter.
- Dimensions are rows $\times$ columns and represented as $r \times c$.
- Element - one of the data entries in a matrix
- Elements are represented by a lower case, italicized letter of the matrix name with subscript indexes.
- Index - a subscript used to indicate the location of an element in terms of its row and column position in a matrix (e.g., $a_{2,3}$ indicates the element located in the $2^{\text {nd }}$ row and $3^{\text {rd }}$ column in matrix $A$.)
- Ex:

- Structure of a matrix
- Row - horizontal arrangement of elements in an array
- Ex:


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## Algebraic Reasoning

Matrix A has 2 rows. The first row, $r_{1}$, has elements $7,-1.5$, and 1. The second row, $r_{2}$, has elements 3,0 , and -7 .

- Column - vertical arrangement of elements in an array
- Ex:

Matrix $A$

$$
\left.\begin{array}{c}
c_{1} \\
{\left[\begin{array}{cc}
C_{2} & c_{3} \\
7 & -1.5 \\
3 & 0
\end{array}\right.} \\
\hline-7
\end{array}\right]
$$

Matrix $A$ has 3 columns. The first column, $c_{1}$, has elements 7 and 3. The second column, $c_{2}$, has elements -1.5 and 0 . The third column, $c_{3}$, has elements 1 and -7 .

- Matrices are represented with a capital, italicized letter and identified by their dimensions, rows $\times$ columns (e.g., matrix $A$ that has 2 rows and 3 columns is described as, "matrix $A$ is a $2 \times 3$ matrix.")
- Ex:

- Scalar - a numerical quantity
- Scalars are similar to scale factors in that they can scale the matrix entries to be greater or less than their original values by a scale factor equivalent to the scalar.
- Multiplication of a scalar, $k$, by a factor matrix generates a product matrix with the same dimensions as the original factor matrix.
- Each element of the product matrix is determined by multiplying the scalar by each corresponding element of the factor matrix.

- Ex:


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## ALgEbRAIC REASONING

Multiply the matrix by the scalar 0.8 in order to multiply each price by $80 \%$.
$0.8\left[\begin{array}{ccc}8.50 & 10.50 & 12.50 \\ 7.95 & 9.95 & 11.95 \\ 8.25 & 10.25 & 12.25 \\ 9.15 & 11.15 & 13.15\end{array}\right]=\left[\begin{array}{lll}0.8(8.50) & 0.8(10.50) & 0.8(12.50) \\ 0.8(7.95) & 0.8(9.95) & 0.8(11.95) \\ 0.8(8.25) & 0.8(10.25) & 0.8(12.25) \\ 0.8(9.15) & 0.8(11.15) & 0.8(13.15)\end{array}\right]=\left[\begin{array}{llc}6.80 & 8.40 & 10.00 \\ 6.36 & 7.96 & 9.56 \\ 6.60 & 8.20 & 9.80 \\ 7.32 & 8.92 & 10.52\end{array}\right]$

The discounted prices of each cup of coffee are shown in the table.

| Item | Small | Medium | Large |
| :---: | :---: | :---: | :---: |
| Holiday Blend | $\$ 6.80$ | $\$ 8.40$ | $\$ 10.00$ |
| Italian Roast | $\$ 6.36$ | $\$ 7.96$ | $\$ 9.56$ |
| Decaffeinated | $\$ 6.60$ | $\$ 8.20$ | $\$ 9.80$ |
| Central American Blend | $\$ 7.32$ | $\$ 8.92$ | $\$ 10.52$ |

- Properties of Operations
- Commutative Property of Scalar Multiplication - interchanging factor matrix and scalar results in the same product matrix
- Ex:

find 3 B and B 3 to determine if the commutative property of scalar multiplication of matrices holds true.
$3\left[\begin{array}{cc}1.5 & 3 \\ -4 & 2.8 \\ 7 & -11\end{array}\right]=\left[\begin{array}{ll}3(1.5) & 3(3) \\ 3(-4) & 3(2.8) \\ 3(7) & 3(-11)\end{array}\right]=\left[\begin{array}{cc}4.5 & 9 \\ -12 & 8.4 \\ 21 & -33\end{array}\right] \quad\left[\begin{array}{cc}1.5 & 3 \\ -4 & 2.8 \\ 7 & -11\end{array}\right] 3=\left[\begin{array}{cc}1.5(3) & 3(3) \\ -4(3) & 2.8(3) \\ 7(3) & -11(3)\end{array}\right]=\left[\begin{array}{cc}4.5 & 9 \\ -12 & 8.4 \\ 21 & -33\end{array}\right]$

Since the product of the scalar times the matrix and the product of the matrix times the scalar are the same, the commutative property of scalar multiplication of matrices holds true.

- Distributive Property of Scalar Multiplication of Matrices Over Addition - if multiplying a scalar by the sum of matrices, the product matrix will be the same as multiplying the scalar to each addend matrix and then adding the scalar multiple matrices together


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## Algebraic Reasoning

- Ex:

Given $A=\left[\begin{array}{ccc}-3 & 2 & 7 \\ -5 & 3 & 10\end{array}\right]$ and $B=\left[\begin{array}{ccc}8 & 4 & 7 \\ 12 & -8 & 5\end{array}\right]$ and scalar 0.6,
find $0.6(A+B)$ and $0.6 A+0.6 B$ to determine if the distributive property of scalar multiplication of matrices over addition holds true.
$0.6(A+B)$


Since the product of the scalar and the matrix are the same, the distributive property of scalar multiplication of matrices over addition holds true.

- Distributive Property of Scalar Multiplication of Matrices Over Subtraction - if multiplying a scalar by the difference of matrices, the product matrix will be the same as multiplying the scalar to the minuend and subtrahend matrix and then subtracting the scalar multiple matrices
- Ex:

$$
\text { Given } A=\left[\begin{array}{ccc}
-3 & 2 & 7 \\
-5 & 3 & 10
\end{array}\right] \text { and } B=\left[\begin{array}{ccc}
8 & 4 & 7 \\
12 & -8 & 5
\end{array}\right] \text { and scalar 0.6, }
$$

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## Algebraic REASONING

find $0.6(A-B)$ and $0.6 A-0.6 B$ to determine if the distributive property of scalar multiplication of matrices over subtraction holds true.


Since the product of the scalar and the matrix are the same, the distributive property of scalar multiplication of matrices over subtraction holds true.

Note(s):

- Grade Level(s):
- Algebra I used the distributive property to multiply a monomial by a polynomial.
- Algebraic Reasoning uses properties of algebra to multiply a scalar by a matrix.
- Algebra II will use properties of algebra to multiply rational expressions.
- Advanced Quantitative Reasoning will use matrix multiplication, including scalars, to solve problems involving arrays of data.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- I.A. Numeric Reasoning - Number representations and operations
- I.A.2. Perform computations with rational and irrational numbers.
- II.A. Algebraic Reasoning - Identifying expressions and equations
- II.A.1. Explain the difference between expressions and equations.


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## Algebraic Reasoning

- II.B. Algebraic Reasoning - Manipulating expressions
- II.B.1. Recognize and use algebraic properties, concepts, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).

Represent and solve systems of two linear equations arising from mathematical and real-world situations using matrices.
Represent, Solve

## SYSTEMS OF TWO LINEAR EQUATIONS ARISING FROM MATHEMATICAL AND REAL-WORLD SITUATIONS USING MATRICES

Including, but not limited to:

- $2 \times 2$ system of linear equations
- Two variables or unknowns
- Two equations
- Standard form for systems of equations have variables on left side of the equal sign in alphabetical order with constant on the right side of the equal sign
- Ex

$$
\begin{aligned}
& 2 x-y=5 \\
& x+y=4
\end{aligned}
$$

- Represent a $2 \times 2$ system of linear equations using matrices.
- Involves coefficient matrix, variable matrix, and constant matrix
$\left.\begin{array}{l}\begin{array}{c}\text { Coefficient } \\ \text { Matrix }\end{array} \\ {\left[\begin{array}{c}\text { Variable } \\ \text { Matrix } \\ \text { of matrix } \\ \text { coefficients }\end{array}\right]}\end{array} \quad \begin{array}{c}\text { Constant } \\ \text { Matrix }\end{array}\right]\left[\begin{array}{c}\left.\begin{array}{c}2 \times 1 \text { matrix } \\ \text { of } \\ \text { variables }\end{array}\right]=\left[\begin{array}{c}2 \times 1 \text { matrix } \\ \text { of } \\ \text { constants }\end{array}\right]\end{array}\right.$
- Ex:

Represent the system of two linear equations with a matrix equation consisting of a coefficient matrix, variable matrix, and constant matrix.

$$
\left\{\begin{array}{r}
2 x-y=5 \\
x+y=4
\end{array}\right.
$$

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## Algebraic Reasoning

Write a matrix equation to represent the system of two linear equations using a coefficient matrix, variable matrix, and constant matrix.

- Place the coefficients in a $2 \times 2$ matrix (column 1 represents the $x$-coefficients and column 2 represents the $y$-coefficients).
- Place the variables in a $2 \times 1$ matrix.
- Place the constants in a $2 \times 1$ matrix after the equal sign.

$$
\left[\begin{array}{rr}
2 & -1 \\
1 & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y
\end{array}\right]=\left[\begin{array}{l}
5 \\
4
\end{array}\right]
$$

- Methods for solving systems of two linear equations in two variables
- Solve matrix equation using inverse operations.
- Inverse matrix - a matrix that can be multiplied by the original matrix to generate the identity matrix
- Inverse matrix is represented with the matrix name and a superscript of -1 and can be calculated using the inverse matrix formula

$$
\text { If } A=\left[\begin{array}{ll}
a & b \\
c & d
\end{array}\right] \text {, then } A^{-1}=\frac{1}{a d-b c}\left[\begin{array}{cc}
d & -b \\
-c & a
\end{array}\right]
$$

- Involves coefficient matrix, variable matrix, and constant matrix
- Solve by left-multiplying both sides of the equation by the inverse matrix.
- Identity matrix - the multiplicative matrix that yields the original matrix when multiplied
- The identity matrix has 1 's in all the diagonal elements of the matrix starting at the top left and 0 's for all of the other elements in the matrix.
- $2 \times 2$ identity matrix

- Using inverse operations with a matrix equation is similar to using inverse operations to solve an equation with real number coefficients and variables.
- Multiplying by the multiplicative inverse will result in the multiplicative identity.
- For real numbers, multiplying by the multiplicative inverse will result in the multiplicative identity of 1 .
- For matrices, multiplying by the multiplicative inverse matrix will result in the identity matrix.
- Systems of two linear equations arising from mathematical situations using inverse operations with a matrix equation
- Ex:


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## Algebraic Reasoning

The number of adults, $x$, and the number of children, $y$, represent a total of 12 people. This equation can be written as

$$
x+y=12
$$

So, the system of equations is

$$
\left\{\begin{array}{c}
5 x+3 y=56 \\
x+y=12
\end{array}\right.
$$

Write a matrix equation to represent the system of two linear equations using a coefficient matrix, variable matrix, and constant matrix.

- Place the coefficients in a $2 \times 2$ matrix (column 1 represents the $x$-coefficients and column 2 represents the $y$-coefficients).
- Place the variables in a $2 \times 1$ matrix.
- Place the constants in a $2 \times 1$ matrix after the equal sign.


Determine the inverse matrix of the coefficient matrix.
Coefficient Matrix


Inverse Matrix

$$
A^{-1}=\frac{1}{(5)(1)-(3)(1)}\left[\begin{array}{cc}
1 & -3 \\
-1 & 5
\end{array}\right]=\frac{1}{2}\left[\begin{array}{rr}
1 & -3 \\
-1 & 5
\end{array}\right]=\left[\begin{array}{rr}
\frac{1}{2} & -\frac{3}{2} \\
-\frac{1}{2} & \frac{5}{2}
\end{array}\right]
$$

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## ALgEBRAIC REASONING

$$
\begin{aligned}
& \text { Solve by left-multiplying both sides of the matrix equation by the inverse of the } \\
& \text { coefficient matrix. } \\
& {\left[\begin{array}{rr}
\frac{1}{2} & -\frac{3}{2} \\
-\frac{1}{2} & \frac{5}{2}
\end{array}\right]\left[\begin{array}{ll}
5 & 3 \\
1 & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y
\end{array}\right]=\left[\begin{array}{rr}
\frac{1}{2} & -\frac{3}{2} \\
-\frac{1}{2} & \frac{5}{2}
\end{array}\right]\left[\begin{array}{l}
56 \\
12
\end{array}\right]} \\
& {\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y
\end{array}\right]=\left[\begin{array}{c}
\frac{1}{2}(56)-\frac{3}{2}(12) \\
-\frac{1}{2}(56)+\frac{5}{2}(12)
\end{array}\right]} \\
& {\left[\begin{array}{l}
x \\
y
\end{array}\right]=\left[\begin{array}{c}
10 \\
2
\end{array}\right]} \\
& \text { The solution to the system of equations }\left\{\begin{array}{c}
5 x+3 y=56 \\
x+y=12
\end{array} \text { is } x=10 \text { and } y=2\right. \\
& \text { There are } 10 \text { adults and } 2 \text { children in the group. }
\end{aligned}
$$

- Inverse operations with a matrix equation using technology
- Involves coefficient matrix, variable matrix, and constant matrix
- Solve using the inverse matrix with technology.
- Systems of two linear equations arising from mathematical situations using inverse operations with a matrix equation using technology
- Ex:

Solve the system of two linear equations using matrices with technology.

$$
\left\{\begin{array}{r}
2 x-y=5 \\
x+y=4
\end{array}\right.
$$

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Algebraic Reasoning
Use the inverse matrix on the home screen of the graphing calculator to compute $[\mathrm{A}]^{-1} \times[\mathrm{C}]$.


Therefore,


The solution to the system of equations


- Systems of two linear equations arising from real-world situations using inverse operations with a matrix equation and technology
- Ex:

A museum charges $\$ 5$ for adult admission and $\$ 3$ for child admission for children aged 5-12 years old. A group of 12 people visits the museum and pays $\$ 56$ for admission. Determine the number of adults and children in the group.
Write a system of linear equations.
Let $x$ represent the number of adults and $y$ represent the number of children.
The price of adult admission is $\$ 5$, so $5 x$ represents the cost of admission for the adults in the group. The price of children's admission is $\$ 3$, so $3 y$ represents the cost of admission for the children in the group. The cost equation can be written
as

$$
5 x+3 y=56
$$

The number of adults, $x$, and the number of children, $y$, represent a total of 12 people. This equation can be written as

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## Algebraic Reasoning <br> $2 \times 3$ matrix <br> of <br> coefficients and constants]



- Augmented matrices can be reduced to row echelon form or reduced row echelon form using Gaussian elimination.
- Row Echelon Form - a matrix where the diagonal elements are all 1s starting at the top left and the elements below a leading 1 are zero
- Ex:

$$
r_{1}\left[\begin{array}{cc|c}
1 & 1 & 4 \\
r_{2} & -3 & -3
\end{array}\right]
$$

- Reduced Row Echelon Form - a matrix where the elements above and below a leading 1 are zero
- Ex:

$$
\begin{array}{ll|l}
r_{1} \\
r_{2}
\end{array}\left[\left.\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array} \right\rvert\, \begin{array}{l}
3
\end{array}\right]
$$

- Gaussian elimination - sequence of elementary row operations on a matrix of coefficients and constants to transform the matrix into row echelon form (ref) or reduced row echelon form (rref)
- Involves manipulating augmented matrix using elementary row operations, including
- Row switching
- Multiplication and/or division of a row by a non-zero number
- Addition and/or subtraction of rows
- Addition and/or subtraction of a multiple of one row with another row
- Manipulate rows to row echelon form and then use substitution to determine solution or manipulate coefficient matrix to reduced row echelon form resulting in a constant matrix that represents the solution
- Systems of two linear equations arising from mathematical situations using an augmented matrix and Gaussian elimination from mathematical situations
- Ex:


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## Algebraic Reasoning

The element in row 2, column 2 needs to be a 1.
To replace row 2 , divide each element in row 2 by -2 ,



The 10 in row 1, column 3 represents the value of $x$, and the 2 in row 2, column 3 represents the value of $y$.

Therefore, $\left\{\begin{array}{l}x=10 \\ y=2\end{array}\right.$
The solution to the system of equations $\left\{\begin{array}{c}5 x+3 y=56 \\ x+y=12\end{array}\right.$ is $x=10$ and $y=2$.
There are 10 adults and 2 children in the group.

- Augmented matrix with technology
- Uses augmented matrices
- Solve using technology to transform the matrix into reduced row echelon form (rref).
- Systems of two linear equations arising from mathematical situations using augmented matrices and technology
- Ex:


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## Algebraic Reasoning

The 3 in row 1, column 3 represents the value of $x$, and the 1 in row 2, column 3 represents the value of $y$.
Therefore, $\left\{\begin{array}{l}x=3 \\ y=1\end{array}\right.$
The solution to the system of equations $\left\{\begin{array}{r}2 x-y=5 \\ x+y=4\end{array}\right.$ is $x=3$ and $y=1$.

- Systems of two linear equations arising from real-world situations using augmented matrices and technology
- Ex:

A museum charges $\$ 5$ for adult admission and $\$ 3$ for child admission for children aged 5-12 years old. A group of 12 people visits the museum and pays $\$ 56$ for admission. Determine the number of adults and children in the group.

Write a system of linear equations.
Let $x$ represent the number of adults and $y$ represent the number of children.
The price of adult admission is $\$ 5$, so $5 x$ represents the cost of admission for the adults in the group. The price of children's admission is $\$ 3$, so $3 y$ represents the cost of admission for the children in the group. The cost equation can be written as

$$
5 x+3 y=56
$$

The number of adults, $x$, and the number of children, $y$, represent a total of 12 people. This equation can be written as

$$
x+y=12
$$

So, the system of equations is

$$
\left\{\begin{array}{c}
5 x+3 y=56 \\
x+y=12
\end{array}\right.
$$

Write an augmented matrix to represent the system of two linear equations.

$$
\begin{array}{ll|l}
r_{1} \\
r_{2}
\end{array}\left[\begin{array}{ll}
5 & 3 \\
1 & 1
\end{array}\right)
$$

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## ALgEBRAIC REASONING

- Algebra I used graphing, tables, and algebraic methods, including substitution and elimination, to solve systems of two linear equations with two variables.
- Algebraic Reasoning introduces the use of matrices to solve systems of two linear equations with two variables
- Algebraic Reasoning extends systems of linear equations to the use of matrices and technology to solve systems of three linear equations with three variables.
- Algebra II will extend systems to three linear equations with three variables and to a linear and a quadratic equation.
- Various mathematical process standards will be applied to this student expectation as appropriate
- TxCCRS:
- I.A. Numeric Reasoning - Number representations and operations
- I.A.2. Perform computations with rational and irrational numbers.
- II.A. Algebraic Reasoning - Identifying expressions and equations
- II.A.1. Explain the difference between expressions and equations
- II.B. Algebraic Reasoning - Manipulating expressions
- II.B.1. Recognize and use algebraic properties, concepts, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).
- II.C. Algebraic Reasoning - Solving equations, inequalities, and systems of equations and inequalities
- II.C.3. Recognize and use algebraic properties, concepts, and algorithms to solve equations, inequalities, and systems of linear equations and inequalities.
- VI.C. Functions - Model real-world situations with functions
- VI.C.1. Apply known functions to model real-world situations.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.3. Determine a solution.
- VII.D. Problem Solving and Reasoning - Real-world problem solving
- VII.D.1. Interpret results of the mathematical problem in terms of the original real-world situation.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
- VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
- IX.A. Connections - Connections among the strands of mathematics
- IX.A.2. Connect mathematics to the study of other disciplines.
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
- IX.B.2. Understand and use appropriate mathematical models in the natural, physical, and social sciences.


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## Algebraic Reasoning

 SYSTEMS OF THREE LINEAR EQUATIONS ARISING FROM MATHEMATICAL AND REAL-WORLD SITUATIONS USING MATRICES AND TECHNOLOGYIncluding, but not limited to:

- $3 \times 3$ system of linear equations
- Three variables or unknowns
- Three equations
- Standard form for linear systems of equations - variables on left side of the equal sign in alphabetical order with constant on the right side of the equal sign
- Ex:

$$
\begin{aligned}
& x+2 y-3 z=-2 \\
& 2 x-2 y+z=7 \\
& 2 x+y+3 z=-4 \\
& \hline
\end{aligned}
$$

- Represent a $3 \times 3$ system of linear equations using matrices
- Involves coefficient matrix, variable matrix, and constant matrix

- Ex:

Represent the system of three linear equations with a matrix equation consisting of a coefficient matrix, variable matrix, and constant matrix.

$$
\left\{\begin{array}{l}
x+2 y-3 z=-2 \\
2 x-2 y+z=7 \\
2 x+y+3 z=-4
\end{array}\right.
$$

Write a matrix equation to represent the system of three linear equations using a coefficient matrix, variable matrix, and constant matrix.

- Place the coefficients in a $3 \times 3$ matrix (column 1 represents the $x$-coefficients, column 2 represents the $y$-coefficients, and column 3 represents the $z$-coefficients).
- Place the variables in a $3 \times 1$ matrix.
- Place the constants in a $3 \times 1$ matrix after the equal sign.


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## Mathematics Enhanced TEKS Clarification Document

## Algebraic REASONING

$$
\left[\begin{array}{ccc}
1 & 2 & -3 \\
2 & -2 & 1 \\
2 & 1 & 3
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{c}
-2 \\
7 \\
-4
\end{array}\right]
$$

- Methods for solving systems of three linear equations in three variables
- Solve the matrix equation using inverse operations.
- Inverse of a matrix - a matrix that can be multiplied by the original matrix to generate the identify matrix
- Inverse matrix is represented with the matrix name and a superscript of -1 and can be calculated using the inverse matrix formula

$$
\text { If } A=\left[\begin{array}{lll}
a & b & c \\
d & e & f \\
g & h & i
\end{array}\right] \text {, then } A^{-1}=\frac{1}{a(e i-f h)-b(d i-f g)+c(d h-e g)}\left[\begin{array}{ccc}
e i-f h & c h-b i & b f-c e \\
f g-d i & a i-c g & c d-a f \\
d h-e g & b g-a h & a e-b d
\end{array}\right]
$$

- Involves coefficient matrix, variable matrix, and constant matrix
- Solve by left-multiplying both sides of the equation by the inverse matrix of the coefficient matrix.
- Identity matrix - the multiplicative matrix that yields the original matrix when multiplied
- The identity matrix has 1 's in all the diagonal elements of the matrix starting at the top left and 0's for all of the other elements in the matrix.
- $3 \times 3$ identify matrix

- Using inverse operations with a matrix equation is similar to using inverse operations to solve an equation with real number coefficients and variables.
- Multiplying by the multiplicative inverse will result in the multiplicative identity.
- For real numbers, multiplying by the multiplicative inverse will result in the multiplicative identify of 1.
- For matrices, multiplying by the multiplicative inverse matrix will result in the identity matrix.
- Systems of three linear equations arising from mathematical situations solved using inverse operations with a matrix equation.
- Ex:

Solve the system of three linear equations using an inverse matrix.

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## Algebraic Reasoning

$$
\begin{gathered}
A^{-1}=\frac{1}{1(-7)-2(4)+(-3)(6)}\left[\begin{array}{ccc}
-7 & -9 & -4 \\
-4 & 9 & -7 \\
6 & 3 & -6
\end{array}\right]=-\frac{1}{33}\left[\begin{array}{ccc}
-7 & -9 & -4 \\
-4 & 9 & -7 \\
6 & 3 & -6
\end{array}\right] \\
A^{-1}=\left[\begin{array}{ccc}
\frac{7}{33} & \frac{3}{11} & \frac{4}{33} \\
\frac{4}{33} & -\frac{3}{11} & \frac{7}{33} \\
-\frac{2}{11} & -\frac{1}{11} & \frac{2}{11}
\end{array}\right]
\end{gathered}
$$

Solve by left-multiplying both sides of the matrix equation by the inverse of the coefficient matrix.


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## Algebraic Reasoning

The solution to the system of equations $\left\{\begin{array}{l}x+2 y-3 z=-2 \\ 2 x-2 y+z=7 \\ 2 x+y+3 z=-4\end{array}\right.$ is $x=1, y=-3$, and $z=-1$.

- Systems of three linear equations arising from real-world situations using inverse matrix
- Ex:

A candy company packages chocolate gift boxes using white chocolate, milk chocolate, and dark chocolate.

- A box of 1 package of white chocolate, 3 packages of milk chocolate, and 3 packages of dark chocolate weighs 14 ounces.
- A box of 1 package of white chocolate, 3 packages of milk chocolate, and 4 packages of dark chocolate weighs 17 ounces.
- A box of 1 package of white chocolate, 4 packages of milk chocolate, and 3 packages of dark chocolate weighs 15 ounces.

Determine the weight of one package of each type of chocolate.

## Write a system of linear equations.

Let $x$ represent the weight of one package of white chocolate, $y$ represent the weight of one package of milk chocolate, and $z$ represent the weight of one package of dark chocolate.

$$
\left\{\begin{array}{l}
x+3 y+3 z=14 \\
x+3 y+4 z=17 \\
x+4 y+3 z=15
\end{array}\right.
$$

Write a matrix equation to represent the system of three linear equations using a coefficient matrix, variable matrix, and constant matrix.

- Place the coefficients in a $3 \times 3$ matrix (column 1 represents the $x$-coefficients, column 2 represents the $y$-coefficients, and column 3 represents the $z$-coefficients).
- Place the variables in a $3 \times 1$ matrix.
- Place the constants in a $3 \times 1$ matrix after the equal sign.


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- Inverse operations with a matrix equation using technology
- Involves coefficient matrix, variable matrix, and constant matrix
- Solve using the inverse matrix with technology.
- Systems of three linear equations arising from mathematical situations using inverse operations with a matrix equation using technology
- Ex:

Solve the system of three linear equations using inverse matrix with technology.

$$
\left\{\begin{array}{l}
x+2 y-3 z=-2 \\
2 x-2 y+z=7 \\
2 x+y+3 z=-4
\end{array}\right.
$$

Write a matrix equation to represent the system of three linear equations using a coefficient matrix, variable matrix, and constant matrix.

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## Algebraic Reasoning

- Place the coefficients in a [3 $\times 3$ ] matrix (column 1 represents the $x$-coefficients, column 2 represents the $y$-coefficients, and column 3 represents the $z$-coefficients).
- Place the variables in a $3 \times 1$ ] matrix.
- Place the constants in a $[3 \times 1]$ matrix after the equal sign.


Enter the coefficients of the $3 \times 3$ matrix into the graphing calculator. (Here matrix $[\mathrm{A}]$ is used.)


Enter the constants of the $3 \times 1$ matrix in the graphing calculator. (Here matrix [C] is used.)


Use the inverse matrix on the home screen of the graphing calculator to compute
$[\mathrm{A}]^{-1} \times[\mathrm{C}]$.

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- Systems of three linear equations arising from real-world situations using inverse matrix with technology
- Ex:

A candy company packages chocolate gift boxes using white chocolate, milk chocolate, and dark chocolate.

- A box of 1 package of white chocolate, 3 packages of milk chocolate, and 3 packages of dark chocolate weighs 14 ounces.
- A box of 1 package of white chocolate, 3 packages of milk chocolate, and 4 packages of dark chocolate weighs 17 ounces.
- A box of 1 package of white chocolate, 4 packages of milk chocolate, and 3 packages of dark chocolate weighs 15 ounces.

Determine the weight of one package of each type of chocolate.
Write a system of linear equations.
Let $x$ represent the weight of one package of white chocolate, $y$ represent the weight of one package of milk chocolate, and $z$ represent the weight of one package of dark chocolate.

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## Algebraic ReAsoning

## $3 \times 4$ matrix <br> of <br> coefficients and constants



The first three columns represents the coefficients.


- Augmented matrices can be reduced to row echelon form or reduced row echelon form using Gaussian elimination.
- Row Echelon Form - a matrix where the diagonal elements are all 1s starting at the top left and the elements below a leading 1 are zero
- Ex:

- Reduced Row Echelon Form - a matrix where the elements above and below a leading 1 are zero
- Ex:

- Gaussian elimination - sequence of elementary row operations on a matrix of coefficients and constants to transform the matrix into row echelon form (ref) or reduced row echelon form (rref)
- Involves manipulating augmented matrix using elementary row operations, including
- Row switching
- Multiplication and/or division of a row by a non-zero number
- Addition and/or subtraction of rows
- Addition and/or subtraction of a multiple of one row with another row


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## Algebraic Reasoning

- Manipulate rows to row echelon form and then use substitution to determine solution or manipulate coefficient matrix to reduced row echelon form resulting in a constant matrix that represents the solution.
- Systems of three linear equations arising from mathematical situations using augmented matrix and Gaussian elimination from mathematical situations
- Ex:

Solve the system of three linear equations shown using an augmented matrix and Gaussian elimination.

$$
\left\{\begin{array}{l}
x+2 y-3 z=-2 \\
2 x-2 y+z=7 \\
2 x+y+3 z=-4
\end{array}\right.
$$

Represent the system with an augmented matrix.

$$
\begin{gathered}
r_{1} \\
r_{2} \\
r_{3}
\end{gathered}\left[\begin{array}{ccc|c}
1 & 2 & -3 & -2 \\
2 & -2 & 1 & 7 \\
2 & 1 & 3 & -4
\end{array}\right]
$$

Using Gaussian elimination, create a matrix in row echelon form.
The element in row 1 , column 1 is already a 1 .
The element in row 2, column 1 needs to be 0.
To replace row 2 , multiply each element in row 3 by -1 and add to each corresponding element in row 2.


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## Algebraic REASONING

Therefore, $\left\{\begin{array}{l}x=1 \\ y=-3 \\ z=-1\end{array}\right.$.
The solution to the system of equations $\left\{\begin{array}{l}x+2 y-3 z=-2 \\ 2 x-2 y+z=7 \\ 2 x+y+3 z=-4\end{array}\right.$ is $x=1, y=-3$, and $z=-1$.

- Systems of three linear equations arising from real-world situations usíng augmented matrix and Gaussian elimination from real-world situations
- Ex:

A candy company packages chocolate gift boxes using white chocolate, milk chocolate, and dark chocolate.

- A box of 1 package of white chocolate, 3 packages of milk chocolate, and 3 packages of dark chocolate weighs 14 ounces.
- A box of 1 package of white chocolate, 3 packages of milk chocolate, and 4 packages of dark chocolate weighs 17 ounces.
- A box of 1 package of white chocolate, 4 packages of milk chocolate, and 3 packages of dark chocolate weighs 15 ounces.

Determine the weight of one package of each type of chocolate.
Write a system of linear equations.
Let $x$ represent the weight of one package of white chocolate, $y$ represent the weight of one package of milk chocolate, and $z$ represent the weight of one package of dark chocolate.

$$
\left\{\begin{array}{l}
x+3 y+3 z=14 \\
x+3 y+4 z=17 \\
x+4 y+3 z=15
\end{array}\right.
$$

Represent the system with an augmented matrix.

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## Algebraic REASONING

Therefore,

$$
\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{c}
1 \\
-3 \\
-1
\end{array}\right] .
$$

The solution to the system of equations $\left\{\begin{array}{l}x+2 y-3 z=-2 \\ 2 x-2 y+z=7 \\ 2 x+y+3 z=-4\end{array}\right.$ is $x=1, y=-3, z=-1$.

- Systems of three linear equations arising from real-world situations using Gaussian elimination and technology
- Ex:

A candy company packages chocolate gift boxes using white chocolate, milk chocolate, and dark chocolate.

- A box of 1 package of white chocolate, 3 packages of milk chocolate, and 3 packages of dark chocolate weighs 14 ounces
- A box of 1 package of white chocolate, 3 packages of milk chocolate, and 4 packages of dark chocolate weighs 17 ounces.
- A box of 1 package of white chocolate, 4 packages of milk chocolate, and 3 packages of dark chocolate weighs 15 ounces.

Determine the weight of one package of each type of chocolate.
Write a system of linear equations.
Let $x$ represent the weight of one package of white chocolate, $y$ represent the weight of one package of milk chocolate, and $z$ represent the weight of one package of dark chocolate.

$$
\left\{\begin{array}{l}
x+3 y+3 z=14 \\
x+3 y+4 z=17 \\
x+4 y+3 z=15
\end{array}\right.
$$

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## Algebraic Reasoning

Therefore, $\left\{\begin{array}{l}x=2 \\ y=1 \\ z=3\end{array}\right.$.
The solution to the system of equations $\left\{\begin{array}{l}x+3 y+3 z=14 \\ x+3 y+4 z=17 \\ x+4 y+3 z=15\end{array}\right.$ is $x=2, y=1$, and $z=3$.
One package of white chocolate weighs 2 ounces, one package of milk chocolate weighs ounce, and one package of dark chocolate weighs 3 ounces.

Note(s):

- Grade Level(s):
- Algebra I used graphing, tables, and algebraic methods, including substitution and elimination, to solve systems of two linear equations with two variables.
- Algebraic Reasoning extends systems of linear equations to the use of matrices and technology to solve systems of three linear equations with three variables.
- Algebra II will extend systems to three linear equations with three variables and to a linear and a quadratic equation.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- I.A. Numeric Reasoning - Number representations and operations
- I.A.2. Perform computations with rational and irrational numbers.
- II.A. Algebraic Reasoning - Identifying expressions and equations
- II.A.1. Explain the difference between expressions and equations.
- II.B. Algebraic Reasoning - Manipulating expressions
- II.B.1. Recognize and use algebraic properties, concepts, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).
- II.C. Algebraic Reasoning - Solving equations, inequalities, and systems of equations and inequalities
- II.C.3. Recognize and use algebraic properties, concepts, and algorithms to solve equations, inequalities, and systems of linear equations and inequalities.
- VI.C. Functions - Model real-world situations with functions
- VI.C.1. Apply known functions to model real-world situations.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.3. Determine a solution.
- VII.D. Problem Solving and Reasoning - Real-world problem solving
- VII.D.1. Interpret results of the mathematical problem in terms of the original real-world situation.


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## Algebraic Reasoning

- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
- VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
- IX.A. Connections - Connections among the strands of mathematics
- IX.A.2. Connect mathematics to the study of other disciplines.
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
- IX.B.2. Understand and use appropriate mathematical models in the natural, physical, and social sciences.

Number and algebraic methods. The student applies mathematical processes to estimate and determine solutions to equations resulting from functions and real-world applications with fluency. The student is expected to:
Estimate a reasonable input value that results in a given output value for a given function, including quadratic, rational, and exponential functions.

Estimate
A REASONABLE INPUT VALUE THAT RESULTS IN A GIVEN OUTPUT VALUE FOR A GIVEN FUNCTION, INCLUDING QUADRATIC, RATIONAL, AND EXPONENTIAL FUNCTIONS

Including, but not limited to:

- Function - a relation in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Input value - elements of the domain of a function
- Input values substitute for the independent variable, $x$, in a function.
- Output value - elements of the range of a function
- Output values substitute for the dependent variable, $y$, or $f(x)$, in a function.
- Estimation of reasonable input value that results in a given output value for a given function
- Tabular analysis
- Select successively smaller intervals of output values to better estimate a reasonable input value that results in the given output value
- Graphical analysis
- Estimate input value, or $x$-coordinate, of point of intersection of horizontal line and graph of given function
- Equation of horizontal line is $y=c$, where $c$ is the given output, or $y$-value
- Point of intersection ( $x, c$ )
- Numerical analysis
- Select successively smaller intervals of output values to better estimate a reasonable input value that results in the given output value
- Bisection method - a numerical method for finding an exact root or estimating a root of a polynomial function by bisecting the interval and checking in which half of the interval the value lies. This method is the average between two points and can be repeated as to determine desired results.


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## Algebraic Reasoning

- Quadratic function
- Ex:

The quadratic function, $h(x)=-16 x^{2}+80 x+3$, describes the height of a ball, $h(x)$, in feet, after it was thrown. Let $x$ represent the amount of time, in seconds, since the ball was thrown. Use a tabular, graphical, and numeric analysis to estimate the time(s) when the ball was at a height of 45 feet.

| Tabular | Graphical |
| :--- | :--- |
| Sample response: | Sample response: |
| The time, in seconds, is the input, $x$. | The time, in seconds, is the input, $x$. The <br> height of the ball, in feet, is the output, |
| The height of the ball, in feet, is the |  | output, $h(x)$.

Use the function $h(x)=-16 x^{2}+80 x+3$ to create a table of values for $h(x)$ to estimate when the ball was at a height of 45 feet. Use the table to estimate the input(s) of the ordered pair, $(x, 45)$.

| Time <br> (seconds), $x$ | Ball Height <br> (feet), $h(x)$ |
| :---: | :---: |
| 0 | 3 |
| 1 | 67 |
| 2 | 99 |
| 3 | 99 |
| 4 | 67 |
| 5 | 3 |

The $y$-value of 45 feet occurs between 0 and 1 seconds, and again between 4 and 5 seconds. Change the interval of the input column to 0.5 seconds to better estimate the input(s), or $x$ coordinate(s), of the ordered pair, $(x$, 45).
height of the ball, in feet, is the output, $h(x)$.
Use the function $h(x)=-16 x^{2}+80 x+3$ to create a graph of $h(x)$ to estimate when the ball was at a height of 45 feet. The height of the ball is the output, so include in the graph the horizontal line $y=45$. Use the graph to estimate the input(s) of the point(s) of intersection, $(x, 45)$.

Height of Ball


Numerical Analysis
Sample response:
The time, in seconds, is the input, $x$. The height of the ball, in feet, is the output, $h(x)$.

Since the path of the ball is parabolic, there should be two times for which the ball is at a height of 45 feet.

Use the bisection method on the function $h(x)=-16 x^{2}+80 x+3$ to determine the first time where the ball is at a height of 45 feet.

Select two $x$-values, such as $x=0$ and $x=2$. Evaluate each in $h(x)$ :
$h(0)=3$
$h(2)=99$
Since $3<45<99$, consider $x=0$ as a lower bound and $x=2$ as an upper bound. Bisect 0 and 2 and use the bisection to evaluate
$h(x)$.
$\frac{0+2}{2}=\frac{2}{2}=1$
$h(0)=3$
$h(1)=67$

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| $\|$Time <br> (seconds), $x$ Ball Height <br> (feet), $h(x)$ <br> 0 3 <br> 0.5 39 <br> 1 67 <br> 1.5 87 <br> 2 99 <br> 2.5 103 <br> 3 99 <br> 3.5 87 <br> 4 67 <br> 4.5 39 <br> 5 3 |
| :---: | :---: | :---: |

The table shows two inputs, 0.5 and 4.5, whose outputs, or $y$-coordinates, are close to 45 . The first output is 39 , slightly less than 45 . Since the $y$-values are increasing, the corresponding input will be slightly more than 0.5 . A reasonable estimate of the first input is 0.6 seconds. The second output is 39, slightly less than 45 . Since the $y$-values are decreasing, the corresponding input will be slightly less than 4.5. A reasonable estimate of the second input is 4.4 seconds. These $x$-values are the inputs that generate an output of 45 feet.

The ball is at a height of 45 feet when it is rising in the air at approximately 0.6 seconds and again when it is falling at approximately 4.4 seconds.

## Algebraic Reasoning

## The graph shows two points of

 intersection. The input of the first point of intersection appears to be slightly more than $x=0.5$. A reasonable estimation of the first input is 0.6 seconds. The input of the second point of intersection appears to be to be slightly less than $x=4.5$. A reasonable estimation of the second input is 4.4 seconds. These $x$-values are the inputs that generate an output of 45 feet.The ball is at a height of 45 feet when it is rising in the air at approximately 0.6 seconds and when it is falling at approximately 4.4 seconds.

Since $3<45<67$, consider $x=1$ as a new upper bound for the interval $x=0$ to $x=1$. Bisect 0 and 1 and use the bisection to evaluate $h(x)$.
$\frac{0+1}{2}=\frac{1}{2}=0.5$
$h(0.5)=39$
$h(1)=67$
Since $39<45<67$, consider $x=0.5$ as a new lower bound for the interval $x=0.5$ to $x=1$. Bisect 0.5 and 1 and use the bisection to evaluate $h(x)$.
$\frac{0.5+1}{2}=\frac{1.5}{2}=0.75$
$h(0.5)=39$
$h(0.75)=54$
Since $39<45<54$, consider $x=0.75$ as a new upper bound for the interval $x=0.5$ to $x=0.75$. Bisect 0.5 and 0.75 and use the bisection to evaluate $h(x)$.
$\frac{0.5+0.75}{2}=\frac{1.25}{2}=0.625$
$h(0.625)=46.75$
$h(0.75)=54$
Since 46.75 is a reasonable estimate of 45 , then a reasonable estimation of the input is 0.625 .

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## Algebraic Reasoning

- Rational functions
- Ex:

Two cities are 330 miles apart. The function $r(t)=\frac{330}{t}$ shows the relationship between the amount of time spent traveling, $t$, in hours, and the speed of travel, $r(t)$, in miles per hour. If the posted speed limit is 75 miles per hour and Celia drives the speed limit, how long will it take Celia to drive between the two cities? Use a tabular, graphical, and symbolic numerical analysis to estimate how long it will take Celia to drive between the two cities.


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## Algebraic Reasoning

The input, or $t$-coordinate, of the ordered pair, ( $t, 75$ ), is exactly 4.4 hours. This $t$-value is the input that generates an output of 75 miles per hour.

If Celia travels at a speed of 75 miles per hour, she will travel the distance between the two cities in exactly 4.4 hours.

- Exponential functions
- Ex:

Herman purchased a new truck for $\$ 18,250$. The truck depreciates, or loses a portion of its value, by $9 \%$ of its value each year. The exponential function $v(t)=18,250(0.91)^{t}$, where $t$ represents time in years, shows the value, $v(t)$, of Herman's truck $t$ years after he purchased it. Use a tabular, graphical, and numerical analysis to determine after how many years Herman's truck will be worth $\$ 10,000$.

| Tabular |  | Numerical Analysis |
| :---: | :---: | :---: |
| Sample response: <br> The time since Herman purchased the truck is the input, $t$. The value of the truck is the output, or dependent variable, $v(t)$. <br> Use the function $v(t)=18,250(0.91)^{t}$ to create a table of values for $v(t)$ to estimate after how many years Herman's truck will be worth $\$ 10,000$. Use the table to estimate the input or $t$-coordinate of the ordered pair, ( $t, 10,000$ ). |  | Sample response: |
|  | The time since Herman purchased the truck is the input, $t$. The value of the truck is the output, $v(t)$. | The time since Herman purchased the truck is the input, $t$. The value of the truck is the output, $v(t)$. |
|  | Use the function $v(t)=18,250(0.91)^{t}$ to create a graph of $v(t)$ on an $x-y$ coordinate plane, where $x$ represents the input variable of time and $y$ represents the output variable of value, to estimate after how many years Herman's truck will be worth $\$ 10,000$. The value of the truck, $\$ 10,000$, is the given output, so include in the graph the horizontal line $y=10,000$. | Use the bisection method on the function $v(t)=18,250(0.91)^{t}$ <br> Select two $t$-values, such as $t=0$ and $t=10$. <br> Evaluate each in $v(t)$ : $\begin{aligned} & v(0)=18,250 \\ & v(10) \approx 7,107 \end{aligned}$ |
| Time  <br> (years), $t$ Value of Truck <br> (dollars), $\nu(t)$ <br>   | Use the graph to estimate the input, or $x$ coordinate, of the point of intersection, ( $x$, | $t=0$ as a lower bound and $t=10$ as an upper bound. Bisect 0 and 10 and use the |
| 0 $18,250.00$ |  |  |

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## Algebraic Reasoning

- Other function families
- Ex:

In clear air, the distance that a lighthouse can be seen from a boat on the water can be calculated using the square root function, $d(x)=1.32 \sqrt{x}$, where $x$ represents the height of the lighthouse in feet. The Port Isabel Lighthouse can be seen from a distance of 10 miles. Use a tabular, graphical, and numerical analysis to determine how tall the lighthouse is.
Tabular
Sample response:
The height of the lighthouse in feet is
the input, $x$. The distance in miles is the the input, $x$. The distance in miles is the output, or dependent variable, $d(x)$.

Use the function $d(x)=1.32 \sqrt{x}$ to create a table of values for $d(x)$ to estimate how tall the Port Isabel Lighthouse is based on a visibility of 10 miles. Use the table to estimate the input or $x$-coordinate of the ordered pair, $(x, 10)$.

| Height of <br> Lighthouse <br> (feet), $x$ | Distance <br> (miles), $d(x)$ |
| :---: | :---: |
| 50 | 9.334 |
| 52 | 9.519 |
| 54 | 9.700 |
| 56 | 9.878 |
| 58 | 10.053 |
| 60 | 10.225 |
| 62 | 10.394 |

The function value of 10 miles occurs between 56 and 58 feet. Change the interval of the input column to 0.1 to

## Sample response:

The height of the lighthouse in feet is the input, $x$. The distance in miles is the output, $d(x)$.

Use the function $d(x)=1.32 \sqrt{x}$ to create a graph of $d(x)$ on an $x$ - $y$ coordinate plane, where $x$ represents the input variable of distance in feet and $y$ represents the output variable of distance in miles, to estimate how tall the Port Isabel Lighthouse is based on a visibility of 10 miles. The distance the Port Isabel Lighthouse can be seen, 10 miles, is the given output, so include in the graph the horizontal line $y=10$. Use the graph to estimate the input or $x$-coordinate of the point intersection, ( $x, 10$ ).

Numerical Analysis
Sample response:
The height of the lighthouse in feet is the input, $x$. The distance in miles is the output, $d(x)$

Use the bisection method on the function, $d(x)=1.32 \sqrt{x}$.

Select two $x$-values, such as $x=50$ and $x=60$. Evaluate each in $d(x)$ :

$$
d(50) \approx 9.33
$$

$$
d(60) \approx 10.22
$$

Since $9.33<10<10.22$, consider $x=50$ as a lower bound and $x=60$ as an upper bound. Bisect 50 and 60 and use the bisection to evaluate $d(x)$.
$\frac{50+60}{2}=\frac{110}{2}=55$
$d(55) \approx 9.79$
Since $9.79<10<10.22$, consider $x=55$ as a new lower bound for the interval $x=55$ and $x=60$. Bisect 55 and 60 and use the bisection to evaluate $d(x)$.

$$
\frac{55+60}{2}=\frac{115}{2}=57.5
$$

$d(57.5) \approx 10.009$

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## Algebraic REASONINg

better estimate the input or $t$-coordinate of the ordered pair $(t, 10)$.

| Height of <br> Lighthouse <br> (feet), $x$ | Distance <br> (miles), $d(x)$ |
| :---: | :---: |
| 57.1 | 9.975 |
| 57.2 | 9.983 |
| 57.3 | 9.992 |
| 57.4 | 10.001 |
| 57.5 | 10.009 |
| 57.6 | 10.018 |
| 57.7 | 10.027 |

The input or $x$-coordinate of the ordered pair ( $x$, 10) appears to be 57.4. A reasonable estimation of the input, or $x$-coordinate, is 57.4 feet. This $x$-value is the input that generates an output of 10 miles.

A lighthouse that can be seen from a distance of 10 miles is approximately 57.4 feet tall. The Port Isabel

Lighthouse is approximately 57.4 feet tall.


The input value or $x$-coordinate of the point of intersection appears to be about halfway between 55 and 60. A reasonable estimation of the input is 57.5 feet. This $x$-value is the input that generates an output of 10 miles.

A lighthouse that can be seen from a distance of 10 miles is approximately 57.5 feet tall. The Port Isabel Lighthouse is approximately 57.5 feet tall.

Since 10.009 is a reasonable estimate of 10 , then a reasonable estimation of the input or $x$-coordinate is 57.5 feet. This $x$-value is the nput that generates an output of 10 miles.

A lighthouse that can be seen from a distance of 10 miles is approximately
57.5 feet tall. The Port Isabel Lighthouse is approximately 57.5 feet tall.

## Note(s):

- Grade Level(s):
- Algebra I determined input values for given output values by solving linear and quadratic equations using graphs, tables, and algebraic methods.
- Algebraic Reasoning solves equations by determining an approximate input value required to generate a desired (or given) output value.
- Algebraic Reasoning solves quadratic equations that model real-world applications tabularly, graphically, and symbolically.
- Algebraic Reasoning uses conceptual methods (e.g., graphs and tables) to estimate a solution to a given rational or exponential equation.
- Algebra II will introduce algebraic methods to solve rational and exponential equations that model real-world applications.


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## Algebraic Reasoning

- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS
- VI.A. Functions - Recognition and representation of functions
- VI.A.2. Recognize and distinguish between different types of functions.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VII.D. Problem Solving and Reasoning - Real-world problem solving
- VII.D.1. Interpret results of the mathematical problem in terms of the original real-world situation.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.

Solve equations arising from questions asked about functions that model real-world applications, including linear and quadratic functions, tabularly, graphically, and symbolically.

Solve

## EQUATIONS ARISING FROM QUESTIONS ASKED ABOUT FUNCTIONS THAT MODEL REAL-WORLD APPLICATIONS, INCLUDING LINEAR

 AND QUADRATIC FUNCTIONS, TABULARLY, GRAPHICALLY, AND SYMBOLICALLYIncluding, but not limited to:

- Function - a relation in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Formulating equations arising from questions asked about functions that model real-world applications
- Identify independent and dependent variables
- Use the given $y$-value or output value from the question to write a related equation from the function
- Methods for solving linear equations that model real-world applications
- Tabular analysis
- Select successively smaller intervals of output values, or $y$-values, to better approximate a solution, or $x$-value, to the stated question
- Graphical analysis
- Approximate input value, or $x$-coordinate, of point of intersection of horizontal line and graph of given function
- Equation of horizontal line is $y=c$, where $c$ is the given output or $y$-value
- Point of intersection ( $x, c$ )
- Symbolic analysis
- Algebraic methods
- Use inverse operations on the related equation to solve for the indicated variable
- Restrictions of solutions in terms of real-world problem situations
- Domain restrictions
- Reasonable solution in context of problem situation
- Ex:


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## Algebraic Reasoning

A bicycle shop charges a $\$ 15$ helmet charge, $\$ 10$ for the first two hours, and $\$ 3.50$ per hour for each subsequent hour for a bicycle rental. The function $c(x)=15+10+3.50(x-2)$ represents the total cost of renting a bicycle for $x$ hours if $x \geq 2$. If Joanna budgeted $\$ 60$ to spend on a bicycle rental, how many hours can she rent the bicycle?

1. Identify the independent and dependent variables represented in the question and the function $c(x)=15+10+3.50(x-2)$.
2. Use the given function value or output value of $\$ 60$ to write a related equation from the function $c(x)$.
3. Determine the solution to the question tabularly, graphically, and symbolically.

Sample response:

1. The independent variable is $x$, the time in hours. The dependent variable is $c(x)$, the cost of renting the bicycle.
2. $c(x)=60$
$c(x)=15+10+3.50(x-2)$
Related equation:
$60=15+10+3.50(x-2)$

| Tabular |  | Symbolic |
| :---: | :---: | :---: |
| Sample response: <br> Use the function $c(x)=15+10+3.50(x-2)$ to create a table of values for $c(x)$ to determine how long Joanna can rent a bike for $\$ 60$. Use the table to determine the $x$-coordinate of the ordered pair, ( $x, 60$ ). | Sample response: <br> Use the function $c(x)=15+10+3.50(x-2)$ to create a graph of $y=c(x)$ on an $x-y$ coordinate plane, where $x$ represents the input variable of time and $y$ represents the output variable of cost, to determine how long Joanna can rent a bike for $\$ 60$. The cost of renting the bike is the output, or $y$-coordinate, so include in the graph the horizontal line $y=60$. Use the graph to determine the $x$-coordinate of the point of intersection, $(x, 60)$. | Sample response: <br> Solve the related equation: $\begin{aligned} & 60=15+10+3.50(x-2) \\ & 60=25+3.50(x-2) \\ & 60=25+3.50 x-7 \\ & 60=18+3.50 x \\ & 60-18=18-18+3.50 x \\ & 42=3.50 x \\ & \frac{42}{3.50}=\frac{3.50 x}{3.50} \\ & 12=x \end{aligned}$ <br> The $x$-value of 12 represents the solution to the related equation $60=15+10+3.50(x-2)$, where $c(12)=60$. <br> For \$60, Joanna can rent a bicycle for 12 hours. |

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|  | Algebraic Reasoning |  |  |
| :---: | :---: | :---: | :---: |
|  | Read across the row to find the value of the solution, or $x$-coordinate, of the ordered pair, $(x, 60) . c(12)$ is exactly 60 , so the solution to the ordered pair $(x, 60)$ is $x=12$ <br> The $x$-value of 12 represents the solution to the related equation $60=15+10+3.50(x-2)$, where $c(12)=60$. <br> For \$60, Joanna can rent a bicycle for 12 hours. |  <br> The $x$-coordinate of the point of intersection appears to be exactly 12 hours. The solution to the ordered pair $(x, 60)$ is $x=12$. <br> The $x$-value of 12 represents the solution to the related equation $60=15+10+3.50(x-2)$, where $c(12)=60$. <br> For $\$ 60$, Joanna can rent a bicycle for 12 hours. <br> Alternatively, use graphing technology to calculate the intersection point between $c(x)=15+10+3.50(x-2)$ and $y=60$, to find ( $x, 60$ ). |  |
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- Methods for solving quadratic equations that model real-world applications
- Tabular analysis
- Select successively smaller intervals of output values, or $y$-values, to better approximate a solution, or $x$-value, to the stated question
- Graphical analysis
- Approximate input value, or $x$-coordinate, of point of intersection of horizontal line and graph of given function
- Equation of horizontal line is $y=c$, where $c$ is the given output, or $y$-value


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## Algebraic Reasoning

- Point of intersection ( $x, c$ )
- Symbolic analysis
- Algebraic methods
- Factoring
- Application of Zero Product Property
- Inverse operations
- Taking square roots
- Completing the square
- Quadratic formula, $\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
- Restrictions of solutions in terms of real-world problem situations
- Domain restrictions
- Reasonable solution in context of problem situation
- Ex:

Joseph drops an object from an initial height of 19.6 meters. The function $h(t)=-4.9 t^{2}+19.6$ represents the height of the object after $t$ seconds. How long until the object hits the ground?

1. Identify the independent and dependent variables represented in the question and the function $h(t)=-4.9 t^{2}+19.6$.
2. Use the given function value or output value of 0 meters to write a related equation from the function $h(t)$.
3. Determine the solution to the question tabularly, graphically, and symbolically.

Sample response:

1. The independent variable is $t$, the elapsed time in seconds. The dependent variable is $h(t)$, the height of the object in meters.
2. $h(t)=0$
$h(t)=-4.9 t^{2}+19.6$
Related equation:
$0=-4.9 t^{2}+19.6$

| Tabular | Graphical | Symbolic |
| :--- | :--- | :--- |
| Sample response: | Sample response: | Sample response: |
| Use the function $h(t)=-4.9 t^{2}+19.6$ to <br> create a table of values for $h(t)$ to determine <br> the time when the object hits the ground. <br> Use the table to determine the $t$-coordinate <br> of the ordered pair, $(t, 0)$.Use the function $h(t)=-4.9 t^{2}+19.6$ to <br> create a graph of $y=h(t)$ on an $x-y$ <br> coordinate plane, where $x$ represents the <br> input variable of time and $y$ represents the | Solve the related equation by using <br> an appropriate factoring method. |  |

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|  | $\|c\|$ <br> Time <br> (seconds), $t$ | Height <br> (meters), $h(t)$ |
| :---: | :---: | :---: |
| 0 | 19.6 |  |
| 1 | 14.7 |  |
| 2 | 0 |  |
| 3 | -24.5 |  |

Read across the row to approximate the value of the solution, or $t$-coordinate, of the ordered pair, $(t, 0) . h(2)$ is 0 , so the solution to the ordered pair $(t, 0)$ is $t=2$.

The $t$-value of 2 represents the solution to the related equation $0=-4.9 t^{2}+19.6$, where $h(2)=0$.

The object dropped from a height of 19.6 meters will hit the ground in 2 seconds.


There are two points of intersection that will satisfy ( $x, 0$ ). The negative $x$-value appears to be at exactly -2 seconds. The positive $x$-value appears to be at exactly 2 seconds. The solutions to the ordered pair $(x, 0)$ are $x=-2$ and $x=2$.

The domain of the function as it models the situation is restricted to positive real numbers, so use the positive solution

Factor -4.9 from the related equation
$0=-4.9 t^{2}+19.6$
$0=-4.9\left(t^{2}-4\right)$

Factor $\left(t^{2}-4\right)$ using an appropriate factor method:

Box method:


The linear factors are $(t+2)$ and $(t-2)$.

Rewrite the related equation using the inear factors.
$0=-4.9\left(t^{2}-4\right)$
$0=-4.9(t+2)(t-2)$
Apply the Zero Product Property to solve for the solution, $t$.
$t+2=0$ and $t-2=0$

$$
t=-2 \text { and } \quad t=2
$$

The domain of the function as it models the situation is restricted to positive real numbers, so use the positive solution, $t=2$ seconds.

The $t$-value of $t=2$ represents the solution to the related equation $0=-4.9 t^{2}+19.6$, where $h(2)=0$

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Use the function $f(x)=\frac{1}{24} x^{2}$ to create a table of values for $f(x)$ to determine the speed of a vehicle that left a skid mark 75 feet long. Use the table to determine the $x$-coordinate, of the ordered pair, ( $x, 75$ ).

| Speed <br> (miles per <br> hour), $x$ | Length of <br> Skid Mark <br> (feet), $f(x)$ |
| :---: | :---: |
| 30 | 37.5 |
| 36 | 54 |
| 42 | 73.5 |
| 48 | 96 |
| 54 | 121.5 |

$f(42)=73.5$ is close to $f(x)=75$. Change the interval of $x$ to 0.1 around 42 to better approximate the solution.

| Speed <br> (miles per <br> hour), $x$ | Length of <br> Skid Mark <br> (feet), $f(x)$ |
| :---: | :---: |
| 42.2 | 74.20 |
| 42.3 | 74.55 |
| 42.4 | 74.91 |
| 42.5 | 75.26 |
| 42.6 | 75.62 |

Read across the row to approximate the value of the solution, or $x$-coordinate, of the ordered pair, ( $x, 75$ ). f(42.4) is approximately 75 , so the solution to the ordered pair $(x, 75)$ is approximately $x=42.4$.

## Algebraic Reasoning

Use the function $f(x)=\frac{1}{24} x^{2}$ to create a graph of $f(x)$ to determine the speed of a vehicle that left a skid mark 75 feet long. The length of the skid mark is the output, or $y$-coordinate, so include in the graph the horizontal line $y=75$. Use the graph to determine the $x$-coordinate of the point of intersection, ( $x, 75$ ).


Speed of Vehicle (miles per hour)
There are two points of intersection whose $x$-coordinates will satisfy $(x, 75)$. The negative $x$-value appears to be about halfway between -45 and -40 . The positive $x$-value appears to be about halfway between the 40 and 45 mph . The solutions to the ordered pair $(x, 75)$ are approximately $x=-42.5$ and $x=42.5$.

Solve the related equation using square roots:

$$
\begin{aligned}
75 & =\frac{1}{24} x^{2} \\
(24) 75 & =\left(\frac{1}{24} x^{2}\right)(24) \\
1800 & =x^{2} \\
\pm \sqrt{1800} & =\sqrt{x^{2}} \\
\pm \sqrt{1800} & =x \\
\pm 30 \sqrt{2} & =x \\
\pm 42.43 & \approx x
\end{aligned}
$$

The domain of the function as it models the situation is restricted to positive real numbers, so use the positive solution, $x \approx 42.43$ seconds. The $x$-value of 42.43 represents the solution to the related equation
$75=\frac{1}{24} x^{2}$, where $f(42.43) \approx 75$.
A vehicle traveling about 42.43 miles per hour should generate a skid mark of 75 feet.

OR
Solve the related equation by using the quadratic formula.

$$
\begin{aligned}
75 & =\frac{1}{24} x^{2} \\
0 & =\frac{1}{24} x^{2}-75
\end{aligned}
$$

Identify $a, b$, and $c$.

$$
a=\frac{1}{24}, b=0, c=-75
$$

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## Algebraic Reasoning

The domain of the function as it models the situation is restricted to positive real numbers, so only $x=42.5$ is a solution to the problem as it is presented in this context.
The $x$-value of 42.5 represents the solution to the related equation $75=\frac{1}{24} x^{2}$, where $f(42.5) \approx 75$.
A vehicle traveling about 42.5 miles per hour should generate a skid mark of 75 feet. Alternatively, use graphing technology to calculate the intersection points, $(x, 75)$.

Substitute values into the quadratic formula,

$$
\begin{aligned}
& x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}, \text { and solve for } w . \\
& x=\frac{-(0) \pm \sqrt{(0)^{2}-4\left(\frac{1}{24}\right)(-75)}}{2\left(\frac{1}{24}\right)} \\
& x=\frac{ \pm \sqrt{\frac{25}{2}}}{\left(\frac{1}{12}\right)} \\
& x= \pm 12 \sqrt{\frac{25}{2}} \\
& x \approx-42.43 \text { and } x \approx 42.43
\end{aligned}
$$

The domain of the function as it models the situation is restricted to positive real numbers, so use the positive solution $x \approx 42.43$ seconds The $x$-value of 42.43 represents the solution to the related equation $75=\frac{1}{24} x^{2}$, where $f(42.43) \approx 75$.

A vehicle traveling about 42.43 miles per hour should generate a skid mark of 75 feet.

- Ex:
A picture frame for an 11 -inch by 14-inch rectangular picture that has a width of $w$ inches. The total area of the frame and picture can be represented by the function $A(w)=(11+2 w)(14+2 w)$, where $w$ represents the width of the frame on each of the four sides. What frame width will generate a total area of 250 square inches?

1. Identify the independent and dependent variables represented in the question and the function $A(w)=(11+2 w)(14+2 w)$.

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## Algebraic Reasoning

2. Use the given function value or output value of 250 square inches to write a related equation from the function $A(w)$.
3. Determine the solution to the question tabularly, graphically, and symbolically.

## Sample response:

1. The independent variable is $w$, the width of the frame on each side of the picture. The dependent variable is $A(w)$, the total area of the frame and picture.
2. $A(w)=250$
$A(w)=(11+2 w)(14+2 w)$
Related equation:
$250=(11+2 w)(14+2 w)$


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## Algebraic Reasoning



$$
\begin{aligned}
& w=\frac{-(50) \pm \sqrt{(50)^{2}-4(4)(-96)}}{2(4)} \\
& w=\frac{-50 \pm \sqrt{2500+1536}}{8} \\
& w=\frac{-50 \pm \sqrt{4036}}{8} \\
& w \approx 1.69 \text { and } w \approx-14.19
\end{aligned}
$$

The domain of the function as it models the situation is restricted to positive real numbers, so only $w \approx 1.69$ is a reasonable solution to the problem as it is presented in this context.

The $w$-value of 1.69 represents an approximate solution to the related equation $250=(11+2 w)(14+2 w)$, where $A(1.69) \approx 250$.

A frame that is approximately 1.69 inches wide will have an area of 250 square inches.

- Methods for solving rational equations (composed of linear and quadratic functions) that model real-world applications
- Tabular analysis
- Select successively smaller intervals of output values, or $y$-values, to better approximate a solution, or $x$-value, to the stated question
- Graphical analysis
- Approximate input value, or $x$-coordinate, of point of intersection of horizontal line and graph of given function
- Equation of horizontal line is $y=c$, where $c$ is the given output, or $y$-value
- Point of intersection ( $x, c$
- Symbolic analysis
- Algebraic methods
- Use inverse operations on related equation to solve for indicated variable
- Restrictions of solutions in terms of real-world problem situations


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## Algebraic Reasoning

- Domain restrictions
- Reasonable solution in context of problem situation
- Ex:

The function $f(w)=\frac{3 \times 10^{8}}{w}$ can be used to determine the frequency, $f(w)$, in hertz, of a radio wave with a wavelength of $w$ meters. If the frequency of an FM radio station is 88.7 MHz (megahertz), what is the wavelength of the radio wave that it broadcasts?

1. Identify the independent and dependent variables represented in the question and the function $f(w)=\frac{3 \times 10^{8}}{w}$.
2. Use the given function value or output value of 88.7 MHz to write a related equation from the function $f(w)$.
3. Determine the solution to the question tabularly, graphically, and symbolically.

Sample response:

1. The independent variable is $w$, the length of the radio wavelength in meters. The dependent variable is $f(w)$, the frequency of the radio wave.
2. $f(w)=88.7 \mathrm{MHz}$
$f(w)=88,700,000 \mathrm{~Hz}$
$f(w)=\frac{3 \times 10^{8}}{w}$
$f(w)=\frac{300,000,000}{w}$
Related equation:
$88,700,000=\frac{300,000,000}{w}$

| Tabular | Graphical | Symbolic |
| :--- | :--- | :--- |
| Sample response: | Sample response: | Sample response: |
| Use the function $f(w)=\frac{300,000,000}{w}$ to | Use the function $f(w)=\frac{300,000,000}{w}$ to | Solve the related equation. |
| create a table of values for $f(w)$ to determine <br> the wavelength of a radio wave broadcast at <br> $88,700,000$ Hz. Use patterns in the table to <br> identify successively smaller intervals of the | create a graph of $y=f(w)$ on an $x-y$ <br> coordinate plane, where $x$ represents the <br> input variable of wavelength and $y$ <br> represents the output variable of frequency, |  |

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## Algebraic Reasoning

dependent values, or function values, to approximate the $w$-coordinate of the ordered pair, ( $w, 88,700,000$ ).

| Wavelength <br> (meters), $w$ | Frequency <br> (hertz), $f(w)$ |
| :---: | :---: |
| 1 | $300,000,000$ |
| 2 | $150,000,000$ |
| 3 | $100,000,000$ |
| 4 | $75,000,000$ |
| 5 | $60,000,000$ |

The function value of $88,700,000 \mathrm{~Hz}$ occurs between 3 and 4 meters. Change the interval of the input column to 0.1 to better determine the solution.

| Wavelength <br> (meters), $w$ | Frequency <br> (hertz), $f(\mathrm{w})$ |
| :---: | :---: |
| 3.1 | $96,774,194$ |
| 3.2 | $93,750,000$ |
| 3.3 | $90,909,091$ |
| 3.4 | $88,235,294$ |
| 3.5 | $85,714,286$ |
| 3.6 | $83,333,333$ |

The function value of $88,700,000 \mathrm{~Hz}$ occurs between 3.3 and 3.4 meters. Change the interval of the input column to 0.01 and then 0.001 to better determine the solution

| Wavelength <br> (meters), $w$ | Frequency <br> (hertz), $f(w)$ |
| :---: | :---: |
| 3.38 | $88,757,396$ |

to determine the wavelength of a radio wave broadcast at $88,700,000 \mathrm{~Hz}$. The wavelength of the radio wave is the output, or $y$-coordinate, so include in the graph the horizontal line $y=88,700,000$. Use the graph to determine the $x$-coordinate of the point of intersection, $(x, 88,700,000)$.


The $x$-coordinate of the point of intersection appears to be slightly less than half-way between the $x$-values 3 and 4 meters. The solution to the ordered pair is $x \approx 3.4$ meters.

The $x$-value of 3.4 represents an approximate solution to the related equation $88,700,000=\frac{300,000,000}{w}$, where $f(3.4) \approx 88,700,000$.

$$
88,700,000=\frac{300,000,000}{w}
$$

$(w) 88,700,000=\left(\frac{300,000,000}{w}\right)(w)$ $88,700,000 w=300,000,000$ $\frac{88,700,000 w}{88,700,000}=\frac{300,000,000}{88,700,000}$

$$
w \approx 3.38
$$

The w-value of 3.38 represents an approximate solution to the related equation
$88,700,000=\frac{300,000,000}{w}$, where
$f(3.38) \approx 88,700,000$.
A radio wavelength of about 3.38 meters will generate a frequency of about 88,700,000 hertz.

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| 5 | 3.381 | $88,731,145$ |
| :---: | :---: | :---: | :---: |
| 3.382 | $88,704,908$ |  |
| 3.383 | $88,678,688$ |  |
| 3.384 | $88,652,482$ |  |
| 3.385 | $88,626,292$ |  |

Read across the row to approximate the value of the solution, or w-coordinate, of the ordered pair, $(w, 88,700,000) . f(3.382)$ is closest to $88,700,000 \mathrm{~Hz}$, so the solution to the ordered pair $(w, 88,700,000)$ is $w \approx 3.382$.

The $w$-value of 3.382 represents an approximate solution to the related equation $88,700,000=\frac{300,000,000}{w}$, where $f(3.382) \approx 88,700,000$.

A radio wavelength of about 3.382 meters will generate a frequency of about 88,700,000 hertz.

Note(s):

- Grade Level(s):
- Algebra I solved linear equations using graphs, tables, and algebraic methods, including direct variation equations.
- Algebra I solved quadratic equations having real solutions using graphs, tables, and algebraic methods, including factoring, taking square roots, completing the square, and applying the quadratic formula.
- Algebraic Reasoning writes related equations from linear and quadratic functions in order to solve a problem and solves the equations using graphs, tables, and algebraic methods.
- Algebra II will introduce absolute value linear equations.
- Algebra II will introduce complex numbers which allow students to solve quadratic equations having non-real solutions.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- II.A. Algebraic Reasoning - Identifying expressions and equations


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## Algebraic ReAsoning

- II.A.1. Explain the difference between expressions and equations.
- II.C. Algebraic Reasoning - Solving equations, inequalities, and systems of equations and inequalities
- II.C.3. Recognize and use algebraic properties, concepts, and algorithms to solve equations, inequalities, and systems of linear equations and inequalities.
- II.D. Algebraic Reasoning - Representing relationships
- II.D.1. Interpret multiple representations of equations, inequalities, and relationships.
- VI.A. Functions - Recognition and representation of functions
- VI.A.2. Recognize and distinguish between different types of functions.
- VI.C. Functions - Model real-world situations with functions
- VI.C.1. Apply known functions to model real-world situations.
- VI.C.2. Develop a function to model a situation.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.3. Determine a solution.
- VII.D. Problem Solving and Reasoning - Real-world problem solving
- VII.D.1. Interpret results of the mathematical problem in terms of the original real-world situation.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
- IX.B.2. Understand and use appropriate mathematical models in the natural, physical, and social sciences.


## Approximate

## SOLUTIONS TO EQUATIONS ARISING FROM QUESTIONS ASKED ABOUT EXPONENTIAL, LOGARITHMIC, SQUARE ROOT, AND CUBIC

 FUNCTIONS THAT MODEL REAL-WORLD APPLICATIONS TABULARLY AND GRAPHICALLYIncluding, but not limited to:

- Function - a relation in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Formulating equations arising from questions asked about functions that model real-world applications
- Identify independent and dependent variables
- Use the given $y$-value or input value from question to write related equation from function
- Approximation of solutions
- Tabular analysis
- Select successively smaller intervals of output values, or $y$-values, to better approximate a solution, or $x$-value, to the stated question


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## Algebraic Reasoning

- Graphical analysis
- Approximate input value, or $x$-coordinate, of point of intersection of horizontal line and graph of given function
- Equation of horizontal line is $y=c$, where $c$ is the given output, or $y$-value
- Point of intersection ( $x, c$ )
- Restriction of solutions
- Domain restrictions
- Reasonable solution in context of problem situation
- Exponential functions that model real-world applications
- Ex:

A coral reef has an area of approximately 14.5 acres and is decreasing in size at a rate of about $8 \%$ each year. The function $A(t)=14.5(0.92)^{t}$ describes the area of the coral reef, $A(t)$, in acres, after $t$ years have passed. How long will it be before the area of the coral reef is 10 acres?

1. Identify the independent and dependent variables represented in the question and the function $A(t)=14.5(0.92)^{t}$.
2. Use the given $y$-value or output value of 10 acres to write a related equation from the function $A(t)$.
3. Approximate the solution to the question tabularly and graphically.

Sample response:

1. The independent variable is $t$, the time in years. The dependent variable is $A(t)$, the area of the coral reef.
2. $A(t)=10$
$A(t)=14.5(0.92)^{t}$
Related equation:
$10=14.5(0.92)^{t}$

| Tabular | Graphical |
| :---: | :---: |
| Sample response: <br> Use the function, $A(t)=14.5(0.92)^{t}$ to create a table of values for $A(t)$ to approximate how long it will take the coral reef to have an area of 10 acres. Use patterns in the table to identify successively smaller intervals of the dependent values, or function values, to approximate the $t$-coordinate of the ordered pair, $(t, 10)$. | Sample response: <br> Use the function $A(t)=14.5(0.92)^{t}$ to create a graph of $y=A(t)$ on an $x-y$ coordinate plane, where $x$ represents the input variable of time and $y$ represents the output variable of area, to approximate how long it will take the coral reef to have an area of 10 acres. The number of acres is the output, or $y$-coordinate, so include in the graph the horizontal line $y=10$. Use the graph to approximate the $x$-coordinate of the point of intersection, $(x, 10)$. |

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## Algebraic Reasoning

The $x$-coordinate of the point of intersection appears to be slightly more than halfway between 160 and 170. The solution to the ordered pair, $(x, 2.3)$ is $x \approx 166$.

The $x$-value of 166 represents an approximate solution to the related equation $2.3=\log (1.2 x)$, where $r(166) \approx 2.3$.

A web page with a page rank of 2.3 has approximately 166 web pages that link to it.

Alternatively, use graphing technology to calculate the intersection point, $(x, 10)$.

- Square root functions that model real-world applications
- Ex:

The period of a pendulum is described by the function $T(x)=2 \pi \sqrt{\frac{x}{9.8}}$, where $x$ represents the length of the pendulum in meters and $T(x)$ is the period of the pendulum in seconds. How long is a pendulum with a period of 2 seconds?

1. Identify the independent and dependent variables represented in the question and the function $T(x)=2 \pi \sqrt{\frac{x}{9.8}}$.
2. Use the given function value or output value of 2 seconds to write a related equation from the function $T(x)$.
3. Approximate the solution to the question tabularly and graphically.

Sample response:

1. The independent variable is $x$, the length of the pendulum in meters. The dependent variable is $r(x)$, the period of the pendulum in seconds
2. $T(x)=2$


Related equation

$$
2=2 \pi \sqrt{\frac{x}{9.8}}
$$

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## Algebraic Reasoning

The $x$-coordinate of the point of intersection is 1 meter. So, the solution to the ordered pair, ( $x, 2$ ), is $x=1$.

The $x$-value of 1 represents the solution to the related equation
$2=2 \pi \sqrt{\frac{x}{9.8}}$, where $T(1) \approx 2$.
A pendulum with a period of 2 seconds has a length of approximately 1 meter.

- Cubic functions that model real-world applications
- Ex:

The electrical power produced by a wind turbine is a function of the speed of the wind. The function $P(v)=1.167 v^{3}$, where $v$ represents the speed of the wind in meters per second and $P(v)$ represents the electrical power, in kilowatts, produced, describes this relationship. How strong of a wind is required to produce 400 kilowatts of electricity?

1. Identify the independent and dependent variables represented in the question and the function $P(v)=1.167 v^{3}$.
2. Use the given function value or output value of 400 kilowatts to write a related equation from the function $P(v)$.
3. Approximate the solution to the question tabularly and graphically.

## Sample response:

1. The independent variable is $x$, the speed of the wind in meters per second. The dependent variable is $P(v)$, the electrical power, in kilowatts, produced
2. $P(v)=400$
$P(v)=1.167 v^{3}$
Related equation:
$400=1.167 v^{3}$

|  | Tabular |
| :--- | :--- |
| Sample response: | Sample response: |

Use the related equation, $400=1.167 v^{3}$, to create a table of values for $P(v)$ to approximate how strong a wind is required to produce 400 kilowatts of electricity. Use patterns in the

Use the function, $P(v)=1.167 v^{3}$, to create a graph of $y=P(v)$ on an $x-y$ coordinate plane, where $x$ represents the input variable of speed and $y$ represents the output variable of power, to

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## Algebraic Reasoning

table to identify successively smaller intervals of the dependent values, or function values, to approximate the $v$-coordinate of the ordered pair, $(v, 400)$

| Wind Speed (meters <br> per second), $v$ | Electrical Power <br> (kilowatts), $P(v)$ |
| :---: | :---: |
| 5 | 145.88 |
| 6 | 252.07 |
| 7 | 400.28 |
| 8 | 597.5 |
| 9 | 850.74 |

$P(7)=400.28$ is close to $P(v)=400$. Change the interval of $v$ to 0.1 around 7 to better approximate the solution.

| Wind Speed (meters <br> per second), $v$ | Electrical Power <br> (kilowatts), $P(v)$ |
| :---: | :---: |
| 6.8 | 366.94 |
| 6.9 | 383.37 |
| 7.0 | 400.28 |
| 7.1 | 417.68 |
| 7.2 | 435.58 |

Read across the row to approximate the value of the solution, or $v$-coordinate, of the ordered pair, $(v, 400)$. $P(7.0)$ is the closest dependent value to 400 , so the solution to the ordered pair $(v, 400)$ is $v \approx 7$.

The $v$-value of 7 represents an approximate solution to the related equation $400=1.167 v^{3}$, where $P(7) \approx 400$.

When the wind is approximately 7 meters per second, 400 kilowatts of electricity are produced.
approximate how strong a wind is required to produce 400 kilowatts of electricity. The electrical power, in kilowatts, produced is the output, or $y$-coordinate, so include in the graph the horizontal line $y=400$. Use the graph to approximate the $x$-coordinate of the point of intersection, ( $x, 400$ ).


Determine the $x$-coordinate that corresponds with the intersection point, ( $x, 400$ ). This $x$-value is the approximated solution to the related equation, $400=1.167 v^{3}$. The $x$-value appears to be 7 meters per second. So, the solution to the ordered pair, $(x, 400)$, is $x \approx 7$.

The $x$-value of 7 represents an approximate solution to the related equation $400=1.167 v^{3}$, where $P(7) \approx 400$.

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## Algebraic Reasoning

When the wind is approximately 7 meters per second, 400 kilowatts of electricity are produced.

Alternatively, use graphing technology to calculate the intersection point, ( $x, 400$ ).

Note(s):

- Grade Level(s):
- Algebra I solved linear and quadratic equations using graphs, tables, and algebraic methods.
- Algebra I made predictions from exponential functions that modeled data from real-world problems.
- Algebraic Reasoning uses the relationship between paired values of the independent and dependent variables so that, when given a value of the dependent variable, students can work backwards to determine the value of the independent variable that would generate it.
- Algebraic Reasoning uses the conceptual understanding of an equation as a specific case of a functional relationship.
- Algebra II will introduce algebraic methods of solving exponential, logarithmic, square root, and cubic equations.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- II.A. Algebraic Reasoning - Identifying expressions and equations
- II.A.1. Explain the difference between expressions and equations.
- II.C. Algebraic Reasoning - Solving equations, inequalities, and systems of equations and inequalities
- II.C.3. Recognize and use algebraic properties, concepts, and algorithms to solve equations, inequalities, and systems of linear equations and inequalities.
- II.D. Algebraic Reasoning - Representing relationships
- II.D.1. Interpret multiple representations of equations, inequalities, and relationships.
- VI.A. Functions - Recognition and representation of functions
- VI.A.2. Recognize and distinguish between different types of functions.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.3. Determine a solution.
- VII.D. Problem Solving and Reasoning - Real-world problem solving
- VII.D.1. Interpret results of the mathematical problem in terms of the original real-world situation.
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
- IX.B.2. Understand and use appropriate mathematical models in the natural, physical, and social sciences.

Modeling from data. The student applies mathematical processes to analyze and model data based on real-world situations with corresponding functions. The student is expected to:

AR.7A Represent domain and range of a function using interval notation, inequalities, and set (builder) notation.
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Algebraic Reasoning

Represent

## DOMAIN AND RANGE OF A FUNCTION USING INTERVAL NOTATION, INEQUALITIES, AND SET (BUILDER) NOTATION

 Including, but not limited to:- Domain and range
- Domain - a set of input values for the independent variable over which the function is defined
- Restricted domain - a set of limited domain values that allows a non-functional relation to become functional
- Continuous function - function whose values are continuous or unbroken over the specified domain
- Discrete function - function whose values are distinct and separate and not connected; values are not continuous. Discrete functions are defined by their domain
- Range - a set of output values for the dependent variable over which the function is defined
- Notation for domain and range
- $\in$ represents an element of a set
- $\mathfrak{R}$ represents the set of real numbers
- $Z$ represents the set of integers
- Q represents the set of rational numbers
- W represents the set of whole numbers
- N represent the set of natural numbers
- Representations of domain and range
- Verbal description
- Interval notation - notation in which the solution is represented by a continuous interval
- Parentheses indicate that the endpoints are open, meaning the endpoints are excluded from the interval.
- Negative infinity, $-\infty$, and positive infinity, $\infty$, are always associated with a parenthesis.
- Brackets indicate that the endpoints are closed, meaning the endpoints are included in the interval
- Inequality notation - notation in which the solution is represented by an inequality statement
- Set (builder) notation - notation in which the solution is represented by a set of values
- Braces are used to enclose the set.
- $\{x \mid x \in$ is read as "The set of $x$ such that $x$ is an element of ..."
- A set could be a list of values contained within braces; e.g., $\{1,2,3,4,5\}$.
- Ex:

| Verbal <br> Description | Interval <br> Notation | Inequality <br> Notation | Set (builder) <br> Notation |
| :--- | :---: | :---: | :---: |
| $x$ is all real numbers <br> less than five | $(-\infty, 5)$ | $x<5, x \in \Re$ | $\{x \mid x \in \Re, x<5\}$ |

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## Algebraic REASONING

|  | Domain | Range |
| :--- | :---: | :---: |
| Verbal | All real numbers except 2 | All real numbers except 1 |
| Interval notation | $(-\infty, 2)$ and $(2, \infty)$ | $(-\infty, 1)$ and $(1, \infty)$ |
| Inequality notation | $x<2$ and $x>2$ | $y<1$ and $y>1$ |
| Set (builder) notation | $\{x \mid x \in \mathfrak{R}, x \neq 2\}$ | $\{y \mid y \in \Re, y \neq 1\}$ |

Note(s):

- Grade Level(s):
- Algebra I represented the domain and range of linear, quadratic, and exponential functions using inequalities and set notation.
- Algebraic Reasoning introduces interval notation.
- Algebraic Reasoning extends the families of functions to include constant, cubic, rational, square root, cube root, logarithmic, and absolute value functions.
- Algebra II reinforces the representation of domain and range of functions in interval, inequality, and set notation.
- Precalculus extends the families of functions to include trigonometric functions and piecewise functions.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- VI.A. Functions - Recognition and representation of functions
- VI.A.2. Recognize and distinguish between different types of functions.
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.

Compare and contrast between the mathematical and reasonable domain and range of functions modeling real-world situations, including linear, quadratic, exponential, and rational functions.

Compare and Contrast
between the mathematical and reasonable domain and range of functions modeling real-workd situations, INCLUDING LINEAR, QUADRATIC, EXPONENTIAL, AND RATIONAL FUNCTIONS

Including, but not limited to:

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## Algebraic Reasoning

- Domain and range
- Domain - a set of input values for the independent variable over which the function is defined
- Restricted domain - a set of limited domain values that allows a non-functional relation to become functional
- Continuous function - function whose values are continuous or unbroken over the specified domain
- Discrete function - function whose values are distinct and separate and not connected; values are not continuous. Discrete functions are defined by their domain.
- Range - a set of output values for the dependent variable over which the function is defined
- Notation for domain and range
- $\in$ represents an element of a set
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- Inequality notation - notation in which the solution is represented by an inequality statement
- Set (builder) notation - notation in which the solution is represented by a set of values
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- $\{x \mid x \in$ is read as "The set of $x$ such that $x$ is an element of ..."
- A set could be a list of values contained within braces; e.g., $\{1,2,3,4,5\}$.
- E
Ex:

| Verbal <br> Description |  |  |  |  | Interval <br> Notation | Inequality <br> Notation | Set (builder) <br> Notation |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x$ is all real numbers <br> less than five | $(-\infty, 5)$ | $x<5, x \in \Re$ | $\{x \mid x \in \Re, x<5\}$ |  |  |  |  |
| $x$ is all real numbers | $(-\infty, \infty)$ | $x \in \Re$ | $\{x \mid x \in \Re\}$ |  |  |  |  |
| $x$ is all whole numbers <br> greater than -3 and <br> less than or equal to 6 | $(-3,6], x \in \mathrm{~W}$ | $-3<x \leq 6, x \in \mathrm{~W}$ | $\{y \mid x \in \mathrm{~W},-3<x \leq 6\}$ |  |  |  |  |

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The mathematical domain of $h(x)$ is all real numbers. The reasonable domain as $h(x)$ models the situation is a subset of the mathematical domain of $h(x)$. The real-world situation restricts the domain to rational numbers, a subset of the real numbers. The situation also restricts the domain to a set of rational numbers that are greater than 0 and are multiples of 0.01 .

The mathematical range of $h(x)$ is all real numbers. The reasonable range as $h(x)$ models the situation is a subset of the mathematical range of $h(x)$. The situation restricts the range to a set of rational numbers that are greater than 0 and are multiples of 0.01 .

- Domain and range of quadratic functions that model real-world situations
- Ex:

A highway descending from a mountain pass has a 10\% grade. The function $d(x)=0.08351 x^{2}-0.001 x$ describes the stopping distance, $d(x)$, in feet, if a recreational vehicle (RV) is driving at a speed of $x$ miles per hour. The maximum speed of the RV is 80 miles per hour. Compare and contrast the mathematical domain and range of $d(x)$ to a reasonable domain and range of $d(x)$ as it models this situation.

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- Domain and range of exponential functions that model real-world situations
- Ex:

Patricia won $\$ 5,000$ from a lottery ticket. She invested the money in an annuity that earns 7.5\% annual interest, compounded annually. The function $b(x)=5000(1.075)^{x}$, where $x$ is the number of years, shows the balance of Patricia's account, $b(x)$, in dollars. Compare and contrast the mathematical domain and range of $b(x)$ to a reasonable domain and range of $b(x)$ as it models this situation.

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- Domain and range of rational functions that model real-world situations
- Ex:

In the United States, the voltage of electrical current that is commercially distributed for home and business use is 110 volts. The rational function $R(x)=\frac{110}{x}$ shows the relationship between $x$, the current in amperes and, $R(x)$, the resistance in ohms, in a circuit using this electrical current. Compare and contrast the mathematical domain and range of $R(x)$ to a reasonable domain and range of $R(x)$ as it models this situation.

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## Algebraic Reasoning

- Algebra I applied linear, quadratic, and exponential functions to model real-world situations.
- Algebra I analyzed linear functions for a reasonable domain and range as it models the situation, including discrete and continuous values.
- Algebraic Reasoning extends analyzing functions for a reasonable domain and range to quadratic, exponential, and rational functions to model real-world situations.
- Algebra Il will apply regression methods available through technology to determine appropriate linear, quadratic, or exponential models for a given data set.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- VI.A. Functions - Recognition and representation of functions
- VI.A.2. Recognize and distinguish between different types of functions.
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
- VIII.A.2. Use mathematical language to represent and communicate the mathematical concepts in a problem.
- VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
- VIII.B.2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.
- VIII.C.3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.

Determine the accuracy of a prediction from a function that models a set of data compared to the actual data using comparisons between average rates of change and finite differences such as gathering data from an emptying tank and comparing the average rate of change of the volume or the second differences in the volume to key attributes of the given model.

## Determine

THE ACCURACY OF A PREDIGTION FROM A FUNCTION THAT MODELS A SET OF DATA COMPARED TO THE ACTUAL DATA USING COMPARISONS BETWEEN AVERAGE RATES OF CHANGE AND FINITE DIFFERENCES SUCH AS GATHERING DATA FROM AN EMPTYING TANK AND COMPARING THE AVERAGE RATE OF CHANGE OF THE VOLUME OR THE SECOND DIFFERENCES IN THE VOLUME TO KEY ATTRIBUTES OF THE GIVEN MODEL

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## Algebraic REASONING

## Including, but not limited to:

- Function - a relation in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Function models determined using finite differences
- Finite difference - a list of differences between the $n^{\text {th }}$ successive $y$-values, $\Delta y$, when differences between first successive $x$-values, $\Delta x$, are constant
- $\Delta x$-change in successive $x$-values (independent values)
- $\Delta y$ - change in successive $y$-values (dependent values)
- Differences in values for successive table rows
- Common difference - a common constant finite difference
- Approximately constant common differences - the average of approximately constant $\Delta y$ differences
- Data patterns that have an approximate first common difference and an approximately constant second common difference with inconsistent signs can be modeled by a linear function.
- Ex:

- Linear function models
- Degree one polynomial function
- Patterns in $y$-values have a common first difference


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## Algebraic Reasoning

- First differences - a list of common differences between the first successive dependent values, $\Delta y$, when first differences between successive independent values, $\Delta x$, are also a common difference
- Approximate constant common differences - the average of approximately constant $\Delta y$ differences
- Data patterns that have an approximate first common difference and an approximately constant second common difference with inconsistent signs can be modeled by a linear function.
- Linear function representations
- Standard form, $a x+b y=c$
- Slope-intercept form, $y=m x+b$, where $m$ represents the slope and $b$ represents the $y$-intercept
- Slope of a linear function is represented by $m=\frac{\text { change in } y \text {-values }}{\text { change in } x \text {-values }}=\frac{\Delta y}{\Delta x}=\frac{\text { first common difference }}{\Delta x}$.
- $(x, y)$ represents a point on the line
- $b$ represents the $y$-intercept, $(0, b)$
- Point-slope form, $y-y_{1}=m\left(x-x_{1}\right)$, where $m$ is the slope and $\left(x_{1}, y_{1}\right)$ represents a point of the line
- Slope of a linear function is represented by $m=\frac{\text { change in } y \text {-values }}{\text { change in } x \text {-values }}=\frac{\Delta y}{\Delta x}=\frac{\text { first common difference }}{\Delta x}$.
- $\left(x_{1}, y_{1}\right)$ represents a point of the line
- Quadratic function models
- Degree two polynomial function
- Patterns in $y$-values have a non-zero common second difference
- Second differences - a list of common differences between the second successive dependent values, $\Delta \mathrm{y}$, when the differences between first successive independent values, $\Delta x$, are also constant.
- Quadratic functions have a dependent second common difference when there is an independent first common difference.
- Approximately constant common differences - the average of approximately constant $\Delta y$ differences
- Data patterns that have an approximate second common difference and an approximately constant third common difference with inconsistent signs can be modeled by a quadratic function.
- Quadratic function representation
- Standard form, $f(x)=a x^{2}+b x+c$, where $a, b$, and $c$ are rational numbers
- First difference between the $y$-values when $x=0$ and $x=1$ is $a+b$.
- Second common differences between the $y$-values are related to a by second common difference is $2 a$ when $\Delta x=1$.
- Value of $c$ represents $(0, c)$ or the $y$-intercept
- Average rate of change - ratio of the difference in function values compared to the difference in input values between two data points
- Average rate of change on an interval $[a, b]$ determined by $\frac{f(b)-f(a)}{b-a}$, where $(a, f(a))$ and $(b, f(b))$ are points from the given data set
- Represents the slope of the line segment connecting points $(a, f(a))$ and $(b, f(b))$ from the given data set
- Predictions using a function model

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## Algebraic Reasoning

- Percent error - measure of how much a predicted value deviates from the actual value; the ratio of the difference between the predicted and actual values compared to the actual value
- Percent Error $=$ predicted value -actual value $\times 100$
- Use function model derived from patterns in finite differences to make prediction
- Calculate percent error between predictions from function model derived from patterns in finite differences and given data set
- Use function model derived from average rates of change to make prediction
- Calculate percent error between predictions from function model derived from average rates of change and given data set
- Use percent error to make comparisons between accuracy of predictions from function model derived from patterns in finite differences and function model derived from average rates of change
- Ex:



## Possible solution:

1. Determine $h(t)$ using finite differences from the table.

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|  | ALgEBRAIC REASONING |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\Delta \boldsymbol{x}$ Independent $\qquad$ First Difference |  |  | $1$ |  | 1 |  |  |
|  |  | Time, t (seconds) | 0 | 1 | 2 | 3 |  | 5 | 6 |
|  |  | Height, $h(t)$ (meters) | 2 | 1.757 | 1.531 | 1.319 |  | 0.944 | 0.780 |
|  |  | $\Delta y$ Depen First Differ | dent ferenc | $+0.017$ | $+0.014$ | $+0.017$ |  | $+0.016$ |  |

The second differences are approximately constant, so write $h(t)$ using a quadratic model in standard form, $h(t)=a t^{2}+b t+c$.
The second differences are equal to $2 a$ since $\Delta x=1$. Determine the average of the second differences and set equal to $2 a$.
$2 a=\frac{0.017+0.014+0.017+0.015+0.016}{5}$
$2 a=0.0158$
$a=0.0079$

Since $h(0)=2, c=2$.
The first difference between $h(0)$ and $h(1)$ is $a+b$, so $a+b=-0.243$.
$a+b=-0.243$
$(0.0079)+b=-0.243$
$b=-0.243-0.0079$
$b=-0.2509$
$h(t)=0.0079 t^{2}-0.2509 t+2$
2. Determine a function, $g(t)$, using the average rate of change in the table.

The average rate of change is the change in the initial and ending $y$-values divided by the change in the initial and ending

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## Algebraic Reasoning

## $x$-values.

average rate of change $=\frac{h(6)-h(0)}{6-0}$
average rate of change $=\frac{0.780-2.0}{6-0}$
average rate of change $=\frac{-1.22}{6}$
average rate of change $\approx-0.2033$
The average rate of change in a linear function is slope, so let $m=-0.2033$.
The $y$-intercept, as seen in the table, is ( 0,2 ), so $b=2$.
$y=m x+b$
$y=(-0.2033) x+(2)$
$g(t)=-0.2033 t+2$
3. Use both functions to predict the time when the tank is completely empty.

The tank will be empty when the function value representing the height of the cylinder is 0 . Use technology to calculate the zero, or $x$-intercept, of each function.


For $h(t)$, there is no zero; however, there is a minimum value near 0 when $t \approx 15.88$ seconds.

For $g(t)$, there is a zero when $t \approx 9.84$ seconds

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## Algebraic Reasoning

4. If the tank actually empties at $t=16$ seconds, determine the accuracy of your predictions from both $g(t)$ and $h(t)$.

How does this value relate to the key attributes of each function?
Use percent error to determine the accuracy of each prediction: Percent Error $=\frac{\text { predicted value }- \text { actual value }}{\text { actual value }} \times 100$.

For $h(t)$, the predicted value was 15.88 seconds and the actual value was 16 seconds.
Percent Error $=\frac{15.88-16}{16} \times 100=\frac{-0.12}{16} \times 100=-0.75 \%$

For $g(t)$, the predicted value was 9.84 seconds and the actual value was 16 seconds.
Percent Error $=\frac{9.84-16}{16} \times 100=\frac{-6.16}{16} \times 100=-38.5 \%$

Both functions predicted a value that was less than the actual value. However, the function $g(t)$, generated by average rates of change, predicted a significantly lower value than what was actually recorded.

The time when the tank is empty represents the $x$-intercept of the function
The time when the tank begins to empty represents the $y$-intercept of the function.

## Note(s):

- Grade Level(s):
- Algebra I used average rates of change (slope) to write linear functions that modeled data.
- Algebraic Reasoning compares average rates of change to finite differences.
- Algebra II will use linear, quadratic, and exponential models for data sets.
- Various mathematical process standards will be applied to this student expectation as appropriate.
- TxCCRS:
- I.B. Numeric Reasoning - Number sense and number concepts
- I.B.1. Use estimation to check for errors and reasonableness of solutions.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VII.D. Problem Solving and Reasoning - Real-world problem solving
- VII.D.2. Evaluate the problem-solving process.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
- VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.


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## Algebraic Reasoning

- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.
- VIII.C.3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.
- IX.A. Connections - Connections among the strands of mathematics
- IX.A.2. Connect mathematics to the study of other disciplines.
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
- IX.B.2. Understand and use appropriate mathematical models in the natural, physical, and social sciences.
- IX.B.3. Know and understand the use of mathematics in a variety of careers and professions.

Determine an appropriate function model, including linear, quadratic, and exponential functions, for a set of data arising from real-world situations using finite differences and average rates of change.

Determine
AN APPROPRIATE FUNCTION MODEL, INCLUDING LINEAR, QUADRATIC, AND EXPONENTIAL FUNCTIONS, FOR A SET OF DATA ARISING FROM REAL-WORLD SITUATIONS USING FINITE DIFFERENCES AND AVERAGE RATES OF CHANGE

Including, but not limited to:

- Data collection using tools such as probes, measurement tools, and software tools
- Data analysis using tools such as spreadsheets and graphing technology
- Function models determined using finite differences
- Finite difference - a list of differences between the $n^{\text {th }}$ successive $y$-values, $\Delta y$, when differences between first successive
$x$-values, $\Delta x$, are constant
- $\Delta x$-change in successive $x$-values (independent values)
- $\Delta y$ - change in successive $y$-values (dependent values)
- Differences in values for successive table rows
- Common difference - a common constant finite difference
- Approximately constant common differences - the average of approximately constant $\Delta y$ differences
- Data patterns that have an approximate first common difference and an approximately constant second common difference with inconsistent signs can be modeled by a linear function.
- Ex:

Real-World Linear Data Patterns

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- Approximately constant common ratios - the average of approximately constant ratios, $\frac{y_{n}}{y_{n-1}}, y_{n-1} \neq 0$, of successive $y$-values, when $\Delta x=1$
- Approximately constant common ratios of successive $y$-values can be modeled by an exponential function.
- Common ratio is the average of approximately constant common successive ratios.
- Second common ratio will have values close to, but inconsistently above or below, the value of one.
- Ex:

Real-World Exponential Data Patterns

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## AlgEBRAIC REASONING



- Linear function models
- Degree one polynomial function
- Patterns in $y$-values have a common first difference
- First differences - a list of common differences between the first successive dependent values, $\Delta y$, when first differences between successive independent values, $\Delta x$, are also a common difference
- Approximately constant common differences - the average of approximately constant $\Delta y$ differences
- Data patterns that have an approximate first common difference and an approximately constant second common difference with inconsistent signs can be modeled by a linear function.
- Linear function representations
- Standard form, $a x+b y=c$
- Slope-intercept form, $y=m x+b$, where $m$ represents the slope and $b$ represents the $y$-intercept


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## Algebraic Reasoning

- Slope of a linear function is represented by $m=\frac{\text { change in } y \text {-values }}{\text { change in } x \text {-values }}=\frac{\Delta y}{\Delta x}=\frac{\text { first common difference }}{\Delta x}$.
- $(x, y)$ represents a point on the line
- b represents the $y$-intercept, $(0, b)$
- Point-slope form, $y-y_{1}=m\left(x-x_{1}\right)$, where $m$ is the slope and ( $x_{1}, y_{1}$ ) represents a point of the line
- Slope of a linear function is represented by $m=\frac{\text { change in } y \text {-values }}{\text { change in } x \text {-values }}=\frac{\Delta y}{\Delta x}=\frac{\text { first common difference }}{\Delta x}$.
- $\left(x_{1}, y_{1}\right)$ represents a point of the line
- Ex:

A research team collected data about the radius of a tree over time, beginning in the year 2010.

| Year Number | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Radius (cm) | 2.5 | 3.1 | 3.5 | 3.9 | 4.4 | 4.9 | 5.5 |

Use patterns in finite differences to determine a function model, $r(x)$, that describes the radius of the tree, in centimeters, in terms of the year number, $x$.

 | $\Delta x$ Independent |
| :--- |
| First Difference | Year Number, $x$ (



The differences in $x$ are all 1 and the first differences in $r(x)$ vary but are close to 0.5 . The first differences are close enough to be considered approximately constant, so determine the average of the first differences to write a linear function rule in slope-intercept form, $y=m x+b$.

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## Algebraic Reasoning

- Quadratic function models
- Degree two polynomial function
- Patterns in $y$-values have a non-zero common second difference
- Second differences - a list of common differences between the second successive dependent values, $\Delta \mathrm{y}$, when the differences between first successive independent values, $\Delta x$, are also constant.
- Quadratic functions have a dependent second common difference when there is an independent first common difference.
- Approximately constant common differences - the average of approximately constant $\Delta y$ differences
- Data patterns that have an approximate second common difference and an approximately constant third common difference with inconsistent signs can be modeled by a quadratic function.
- Quadratic function representation
- Standard form, $f(x)=a x^{2}+b x+c$, where $a, b$, and $c$ are rational numbers
- First difference between the $y$-values when $x=0$ and $x=1$ is $a+b$.
- Second common differences between the $y$-values are related to a by second common difference is $2 a$ when $\Delta x=1$.
- Value of $c$ represents $(0, c)$ or the $y$-intercept
- Ex:

A cylindrical tank with a radius of 0.5 meters and a height of 2 meters is completely filled of water. The tank drains through a hole with a 0.1 meter radius in the bottom of the tank. The table shows the height of the water, $h$, inside the tank from the bottom of the tank, $t$ seconds after the stopper in the hole is removed. Use patterns in finite differences to determine, $h(t)$, that describes the height of the water level, $h(t)$, in terms of the elapsed time in seconds, $t$.

| Time, $t$ <br> $($ seconds) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Height, $h(t)$ <br> (meters) | 2 | 1.757 | 1.531 | 1.319 | 1.124 | 0.944 | 0.780 |



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## Algebraic Reasoning

Use patterns in finite differences to determine, $h(t)$, that describes the height of the water level, $h(t)$, in terms of the elapsed time in seconds, $t$.


The second differences are approximately constant, so write $h(t)$ using a quadratic model in standard form, $h(t)=a t^{2}+b t+c$.
The second differences are equal to $2 a$ since $\Delta x=1$. Determine the average of the second differences and set equal to $2 a$.
$2 a=\underline{0.017+0.014+0.017+0.015+0.016}$
$2 a=0.0158$
$a=0.0079$
Since $h(0)=2, c=0$.
The first difference between $h(0)$ and $h(1)$ is $a+b$, so $a+b=-0.243$
$a+b=-0.243$
$(0.0079)+b=-0.243$
$b=-0.243-0.0079$
$b=-0.2509$
$h(t)=0.0079 t^{2}-0.2509 t+2$

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## Algebraic ReAsoning

Graph $h(t)$ and create a scatterplot of the data to compare the quadratic function model to the given data set.


The graph shows the quadratic function model, $h(t)$, approximates the data set closely, so $h(t)=0.0079 t^{2}-0.2509 t+2$ is an appropriate function model for the data set.

- Exponential function models
- Patterns in $y$-values differences have a common absolute value resulting in a common ratio
- Exponential functions have successive $y$-value differences that result in a common ratio when there is an independent first common difference.
- Approximately constant common ratios - the average of approximately constant ratios, $\frac{y_{n}}{y_{n-1}}, y_{n-1} \neq 0$, of successive $y$-values, when $\Delta x=1$
- Approximately constant common ratios of successive $y$-values can be modeled by an exponential function.
- Common ratio is the average of approximately constant common successive ratios.
- Second common ratio will have values close to, but inconsistently above or below, the value of one.
- Exponential function representation
- Standard form, $f(x)=a b^{x}$, where a represents the initial value, or the $y$-intercept $(0, a)$, and $b$ represents the base of the exponential function, or the common ratio
- Ex:


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## Algebraic Reasoning

Isabella dropped a properly inflated basketball from a height of 180 centimeters above the ground. She used technology to measure the height of the first six bounces and recorded her data in a table. Determine a function model that describes $h(x)$, the bounce height in centimeters, when $x$ represents the number of bounces since Isabella dropped the basketball.


The differences in $x$ are all 1 and the successive ratios in $h(x)$ vary but are close to 0.75 . They are close enough to be considered approximately constant, so determine the average of the successive ratios to write the function rule in the form $h(x)=a b^{x}$
$b=\frac{0.75+0.74+0.75+0.73+0.73+0.75}{6}$
$b=\frac{4.45}{6}$
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## ALgEBRAIC REASONING

- Average rate of change on an interval $[a, b]$ determined by $\frac{f(b)-f(a)}{b-a}$, where (a, f(a)) and (b,f(b)) are points from the given data set
- Represents the slope of the line segment connecting points (a,f(a)) and (b,f(b)) from the given data set
- Ex:

A research team collected data about the radius of a tree over time, beginning in the year 2010. Determine a function model, $r(x)$ that describes the radius of the tree, in centimeters, in terms of the year number, $x$.

| Year | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year Number, $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| Radius (cm), $r(x)$ | 2.5 | 3.1 | 3.5 | 3.9 | 4.4 | 4.9 | 5.5 |

Use points $(0,2.5)$ and $(6,5.5)$ to determine the average rate of change over the interval $[0,6]$.
$(a, r(a))=(0,2.5)$
$(b, r(b))=(6,5.5)$
$\frac{r(b)-r(a)}{b-a}=\frac{5.5-2.5}{6-0}$
$\frac{r(b)-r(a)}{b-a}=\frac{3}{6}$
$\frac{r(b)-r(a)}{b-a}=\frac{1}{2}$
The $y$-intercept occurs when $x=0$. From the table, $r(0)=2.5$, so the $y$-intercept is $b=2.5$.
Use the average rate of change as the slope, $m$, and the $y$-intercept, $b$, to write $r(x)$ in slope-intercept form.
$y=m x+b$
$r(x)=\left(\frac{1}{2}\right) x+$
$r(x)=\frac{1}{2} x+2.5$
Graph $r(x)$ and create a scatterplot of the data to compare the linear function model to the given data set.

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## ALgebraic Reasoning

- VI.A.2. Recognize and distinguish between different types of functions.
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions.
- VI.C. Functions - Model real-world situations with functions
- VI.C.1. Apply known functions to model real-world situations.
- VI.C.2. Develop a function to model a situation.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VII.A.2. Formulate a plan or strategy.
- VII.D. Problem Solving and Reasoning - Real-world problem solving
- VII.D.1. Interpret results of the mathematical problem in terms of the original real-world situation.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
- VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
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- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
- IX.B.2. Understand and use appropriate mathematical models in the natural, physical, and social sciences.
- IX.B.3. Know and understand the use of mathematics in a variety of careers and professions.

Determine

## IF A GIVEN LINEAR FUNCTION IS A REASONABLE MODEL FOR A SET OF DATA ARISING FROM A REAL-WORLD SITUATION

Including, but not limited to:

- Function - a relation in which each element of the domain $(x)$ is paired with exactly one element of the range $(y)$
- Reasonableness of a given linear function model for a given data set
- Comparison of predictions using given línear function model and the given data set
- Percent error - measure of how much a predicted value deviates from the actual value; the ratio of the difference between the predicted and actual values compared to the actual value


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## Algebraic REASONING

- Percent Error = predicted value -actual value


## - Ex:

The table shows the population of Detroit, Michigan, for each decade since 1950. Determined using average rates of change, the function $p(x)=1,849,568-189,298.5 x$ could model the population data where $x$ is the decade since 1950 and $p(x)$ is the population of Detroit. Is $p(x)$ a reasonable linear model for this data? Explain your reasoning.

| Decade <br> since <br> 1950 | Year | Population |
| :---: | :---: | :---: |
| 0 | 1950 | $1,849,568$ |
| 1 | 1960 | $1,670,144$ |
| 2 | 1970 | $1,514,063$ |
| 3 | 1980 | $1,203,368$ |
| 4 | 1990 | $1,027,974$ |
| 5 | 2000 | 951,270 |
| 6 | 2010 | 713,777 |

## Sample response

Use technology to generate a table of values for $p(x)$. Compare the function values to the actual data values using percent error.

| Decade <br> since <br> $1950, x$ | Year | Population | $p(x)$ | Percent Error <br> $\frac{p(x)-\text { population }}{\text { population }} \times 100$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1950 | $1,849,568$ | $1,849,568$ | 0 |
| 1 | 1960 | $1,670,144$ | $1,660,270$ | $-0.59 \%$ |
| 2 | 1970 | $1,514,063$ | $1,470,971$ | $-2.85 \%$ |
| 3 | 1980 | $1,203,368$ | $1,281,673$ | $6.51 \%$ |
| 4 | 1990 | $1,027,974$ | $1,092,374$ | $6.26 \%$ |
| 5 | 2000 | 951,270 | 903,076 | $-5.07 \%$ |
| 6 | 2010 | 713,777 | 713,777 | $0 \%$ |

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## Algebraic Reasoning

The function values for $p(x)$ are within at most $6.5 \%$ of the actual population values. The average percent error is $0.61 \%$, so $p(x)$ is a reasonable linear model for this data set.

- Ex:

The table shows the population of Detroit, Michigan, for each decade between 1820, the first time the US Census was taken for Detroit after Michigan became a state, and 1920. Determined using average rates of change, the function $q(x)=1,422+99,225.6 x$ could model the population data, where $x$ is the decade since 1920 and $q(x)$ is the population of Detroit. Is $q(x)$ a reasonable model for this data? Explain your reasoning.

| Year | Decade <br> since <br> 1820 | Population |
| :---: | :---: | :---: | :---: |
| 1820 | 0 | 1,422 |
| 1830 | 1 | 2,222 |
| 1840 | 2 | 9,102 |
| 1850 | 3 | 21,019 |
| 1860 | 4 | 45,619 |
| 1870 | 5 | 79,577 |
| 1880 | 6 | 116,340 |
| 1890 | 7 | 205,876 |
| 1900 | 8 | 285,704 |
| 1910 | 9 | 465,766 |
| 1920 | 10 | 993,678 |

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## Algebraic ReAsoning

Possible solution: Use technology to generate a table of values for $q(x)$. Compare the function values to the actual data values using percent error.
\(\left.$$
\begin{array}{|c|c|c|c|c|}\hline \begin{array}{c}\text { Decade } \\
\text { since } \\
1820, x\end{array}
$$ \& Year \& Population \& q(x) \& \begin{array}{c}Percent Error <br>
q(x)-polulation <br>

population\end{array} \times 100\end{array}\right]\)| $0 \%$ |
| :---: |
| 0 |

The percent errors for $q(x)$ vary greatly and in most cases are well over $100 \%$. The linear function, $q(x)$, is not a reasonable model for this data set.

- Comparison of regression equation created from data set and the given linear function model
- Regression equation - line of best fit representing a set of bivariate data
- Correlation coefficient (r-value) - numeric value that assesses the strength of the linear relationship between two quantitative variables in a set of bivariate data
- The regression equation and correlation coefficient for a given set of data can be computed using graphing technology.
- Correlation for approximated $r$-values
- Weak, very weak, to no correlation as it approaches 0 : $0 \leq|r|<0.33$
- Moderate correlation: $0.33 \leq|r|<0.67$
- Strong, very strong: $0.67 \leq|r|<1.00$
- Data form a perfect line: $r= \pm 1$
- Compare the constants and coefficients in the linear regression model to the given linear model for the data set
- Ex:


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## Algebraic Reasoning

The table shows the population of Detroit, Michigan, for each decade since 1950. Determined using average rates of change, the function $p(x)=1,849,568-189,298.5 x$ could model the population data where $x$ is the decade since 1950 and $p(x)$ is the population of Detroit. Is $p(x)$ a reasonable linear model for this data? Explain your reasoning.

| Decade <br> since <br> 1950 | Year | Population |
| :---: | :---: | :---: |
| 0 | 1950 | $1,849,568$ |
| 1 | 1960 | $1,670,144$ |
| 2 | 1970 | $1,514,063$ |
| 3 | 1980 | $1,203,368$ |
| 4 | 1990 | $1,027,974$ |
| 5 | 2000 | 951,270 |
| 6 | 2010 | 713,777 |

Use technology to determine a linear regression function. Define the independent variable, $x$, as the decade since 1950, and the dependent variable, $y$, as the population.


The linear regression equation is $y=-190,400.36 x+1,846,938.79$. The constant term, $1,846,928.79$, is close to the constant term in the linear function model $p(x), 1,849,568$. The coefficient of the linear term, $-190,400.36$, is close to the coefficient of the linear term in the linear function model $p(x),-189,298.5$. The correlation coefficient, or $r$-value, is 0.99 , which indicates the linear regression equation has a very strong linear correlation for the given data.

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## Algebraic Reasoning

Because the constant terms and coefficients of the linear terms are reasonably close in value, and the linear regression equation shows a very strong linear correlation, the function $p(x)$ is a reasonable linear model for this data.

- Comparison of graph of given linear function model and scatterplot of given data set
- Ex:

The table shows the population of Detroit, Michigan, for each decade since 1950. Determined using average rates of change, the function $p(x)=1,849,568-189,298.5 x$ could model the population data where $x$ is the decade since 1950 and $p(x)$ is the population of Detroit. Is $p(x)$ a reasonable linear model for this data? Explain your reasoning.

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Graph $p(x)$ and create a scatterplot of the data to compare the linear function model to the given data set.

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## Algebraic Reasoning

- VI.A. Functions - Recognition and representation of functions
- VI.A.2. Recognize and distinguish between different types of functions.
- VI.B. Functions - Analysis of functions
- VI.B.1. Understand and analyze features of functions.
- VI.C. Functions - Model real-world situations with functions
- VI.C.1. Apply known functions to model real-world situations.
- VI.C.2. Develop a function to model a situation.
- VII.A. Problem Solving and Reasoning - Mathematical problem solving
- VII.A.1. Analyze given information.
- VII.A.2. Formulate a plan or strategy.
- VII.D. Problem Solving and Reasoning - Real-world problem solving
- VII.D.1. Interpret results of the mathematical problem in terms of the original real-world situation.
- VIII.A. Communication and Representation - Language, terms, and symbols of mathematics
- VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
- VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
- VIII.B. Communication and Representation - Interpretation of mathematical work
- VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
- VIII.C. Communication and Representation - Presentation and representation of mathematical work
- VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.
- IX.A. Connections - Connections among the strands of mathematics
- IX.A.2. Connect mathematics to the study of other disciplines.
- IX.B. Connections - Connections of mathematics to nature, real-world situations, and everyday life
- IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
- IX.B.2. Understand and use appropriate mathematical models in the natural, physical, and social sciences.
- IX.B.3. Know and understand the use of mathematics in a variety of careers and professions.


## Bibliography:

Texas Education Agency \& Texas Higher Education Coordinating Board. (2015). Texas college and career readiness standards. Retrieved from http://www.thecb.state.tx.us/collegereadiness/crs.pdf

Texas Education Agency. (2016). Mathematics TEKS - supporting information algebraic reasoning. Retrieved from https://www.texasgateway.org/resource/mathematics-teks-supporting-information

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